

Mumford–Shah Regularized DBSCAN for Unsupervised Image Segmentation

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Abstract

Image segmentation is a fundamental pre-processing task in computer vision, enabling region-based analysis for a wide range of applications. Density-based clustering methods such as DBSCAN naturally preserve spatial connectivity and can identify arbitrarily shaped regions; however, when applied directly to image data, they often suffer from parameter sensitivity and noisy segmentation results due to the lack of explicit boundary modeling. Conversely, variational approaches such as the Mumford–Shah model produce coherent regions with smooth boundaries but are computationally expensive for large images. This paper proposes a hybrid image segmentation framework that integrates Mumford–Shah-inspired boundary regularization into a density-based DBSCAN clustering process. Boundary information derived from image gradients is incorporated into the neighborhood expansion mechanism to discourage cluster growth across strong edges. In addition, an automatic region merging strategy is applied to reduce over-segmentation and improve region consistency. The proposed method avoids the computational overhead of full variational optimization while retaining essential boundary-preserving properties. Experimental results on natural and textured images demonstrate that the proposed approach produces more coherent and boundary-aligned segmentations than standard and adaptive DBSCAN variants, while maintaining practical computational efficiency.

Keywords: *Image Segmentation; Density-Based Clustering; DBSCAN; Mumford–Shah Model; Boundary Regularization; Unsupervised Learning; Spatial Connectivity; Variational Methods; Computer Vision.*

1. INTRODUCTION

Image segmentation serves as a core pre-processing step in computer vision, aiming to partition an image into homogeneous and spatially connected regions of interest [1–3]. These regions provide the foundation for a wide range of applications, including satellite image analysis, biometric identification, and clinical diagnostics [3,30]. Despite extensive research, the absence of a general-purpose segmentation method capable of producing consistent results across diverse image types remains a major challenge, motivating continued investigation in this area [1,2]. Clustering-based segmentation techniques are widely adopted due to their conceptual simplicity and computational efficiency [4].

Methods such as K-means and its variants have been extensively studied for image segmentation [5–7], but their reliance on color similarity often leads to fragmented regions and poor boundary localization, particularly in textured or noisy images [18]. Several extensions incorporating spatial constraints, adaptive penalties, and regularization terms have been proposed to mitigate these issues [8–10]; however, these methods still struggle to enforce strong spatial connectivity and accurate boundary preservation.

Density-based clustering algorithms, most notably DBSCAN, offer an alternative paradigm by naturally encoding spatial connectivity and allowing the detection of arbitrarily shaped regions without requiring the number of clusters to be predefined [15]. DBSCAN-based image segmentation approaches have shown promise in preserving connected regions [16–19], yet they remain sensitive to parameter selection and lack explicit boundary modeling, often resulting in noisy or over-segmented outputs when applied directly to pixel-level image data [17,18].

Variational methods such as the Mumford–Shah model address these limitations by formulating segmentation as an energy minimization problem that enforces region homogeneity while penalizing excessive boundary length [20,21]. Although Mumford–Shah-based methods produce smooth and well-defined boundaries, their computational cost limits their applicability to large-scale images [22,26].

Recent work has demonstrated that integrating Mumford–Shah regularization into clustering frameworks such as K-means and Pairwise Nearest Neighbor (PNN) can significantly improve segmentation quality while maintaining computational efficiency [27–29]. Motivated by these findings, this work proposes a hybrid framework that embeds Mumford–Shah-inspired boundary regularization within a density-based DBSCAN process, combined with automatic region merging to reduce over-segmentation and improve region coherence.

2. RELATED WORK

Image segmentation techniques can broadly be categorized into clustering-based, density-based, and variational approaches [1,2]. Among clustering-based methods, K-means and Fuzzy C-means algorithms have been widely applied due to their efficiency and simplicity [5,8]. Numerous extensions incorporating spatial constraints and adaptive regularization have been proposed to improve segmentation performance [9,10,18], though these methods often require predefined cluster numbers and remain prone to region fragmentation.

Hierarchical and agglomerative clustering approaches, including Ward’s method and Pairwise Nearest Neighbor (PNN) algorithms, have also been explored for image segmentation [11–14]. While hierarchical clustering provides flexible region formation, it can be computationally expensive. Significant research has focused on accelerating these methods through efficient data structures and neighborhood graph optimizations [13,14].

Density-based clustering algorithms, particularly DBSCAN, have gained attention for image segmentation due to their ability to preserve spatial connectivity and detect arbitrarily shaped regions [15]. DBSCAN has been successfully applied to color image segmentation, superpixel generation, and multispectral imagery [16–19]. However, traditional DBSCAN lacks explicit boundary preservation mechanisms, making it sensitive to noise and parameter selection [17,18].

Variational segmentation models, most notably the Mumford–Shah functional, provide a mathematically principled approach to segmentation by minimizing an energy function that balances data fidelity and boundary regularity [20,21]. Numerous approximations, convex relaxations, and fast optimization strategies have been proposed to reduce computational complexity [22–26]. Despite these improvements, pure variational approaches remain computationally demanding.

Hybrid approaches integrating variational models with clustering techniques have recently gained traction. In particular, Shah and Fränti demonstrated that incorporating the Mumford–Shah model into K-means and PNN frameworks leads to improved segmentation quality with reduced computational cost [27–29]. Building on this line of research, the present work explores a novel integration of Mumford–Shah-inspired boundary regularization into a density-based DBSCAN framework, aiming to combine spatial connectivity, boundary preservation, and computational efficiency.

Classical unsupervised segmentation methods can be broadly divided into clustering-based, graph-based, and variational approaches. Clustering techniques such as K-means and fuzzy C-means group pixels based on feature similarity but often neglect spatial coherence, leading to fragmented regions [2], [9], [15]. Graph-based methods, including normalized cuts, model images as weighted graphs and achieve segmentation through global optimization, but their computational complexity limits scalability [47].

2.1 K-means and Its Variants

K-means clustering remains one of the most widely used hard clustering methods for image segmentation due to its simplicity and computational efficiency [9]. However, its practical application is hindered by the requirement to predefine the number of clusters and its sensitivity to initialization, which can lead to suboptimal segmentation results [11]. Numerous improvements have been proposed, including adaptive objective functions [12], improved initialization and repetition strategies [13], and random swap mechanisms to escape poor local minima [14]. Additional extensions incorporate spatial constraints and regularization to improve region consistency [15]–[18].

2.2 Agglomerative Clustering and PNN Methods

Agglomerative clustering offers a hierarchical alternative to K-means, beginning with each pixel as an individual cluster and iteratively merging the most similar pairs [30], [31]. Among these methods, the Pairwise Nearest Neighbor (PNN) algorithm and its Ward-based variants minimize information loss during merging and do not require the number of clusters to be specified in advance [32], [33]. Despite these advantages, classical PNN approaches are computationally expensive for large images.

To address this limitation, several accelerated variants have been proposed, including fast exact PNN, Lazy-PNN, and k-nearest neighbor graph-based methods [36], [38], [42]. Iterative shrinking further improves clustering quality by reassigning removed cluster elements to nearby clusters rather than enforcing strict pairwise merges [41]. While these methods significantly enhance efficiency, they still lack explicit boundary modeling, motivating hybrid approaches that integrate variational regularization.

2.3 Density-Based Spatial Clustering (DBSCAN)

Density-Based Spatial Clustering of Applications with Noise (DBSCAN) is a widely used unsupervised clustering algorithm that identifies clusters as contiguous regions of high point density separated by low-density areas [32]. Unlike partition-based methods such as K-means, DBSCAN does not require the number of clusters to be specified a priori and is capable of discovering arbitrarily shaped clusters while explicitly identifying noise points.

Let $\mathbf{x}_i \in \mathbb{R}^d$ denote the feature vector associated with pixel i , incorporating spatial coordinates and appearance attributes.

The ε -neighborhood of x_i is defined as

$$N\varepsilon(x_i) = \{ x_j \mid \|x_i - x_j\| \leq \varepsilon \} \quad (1)$$

A point x_i is a core point if the number of points in its neighborhood satisfies

$$|N\varepsilon(x_i)| \geq \text{minPts} \quad (2)$$

Clusters are formed by recursively aggregating density-reachable points, where a point x_i is density-reachable from x_j if there exists a chain of points such that each lies within the ε -neighborhood of a core point [32].

In image segmentation, DBSCAN naturally preserves spatial connectivity when pixel coordinates are embedded in the feature space, enabling the detection of irregularly shaped regions [26], [27]. However, its performance is highly sensitive to the choice of ε and minPts. Inappropriate parameter selection often results in noisy segmentation, fragmented regions, or excessive merging across object boundaries [28], [29]. Moreover, standard DBSCAN lacks an explicit mechanism to model image boundaries, making it prone to leakage across strong edges.

To address computational efficiency, several accelerated DBSCAN variants have been proposed. Li et al. [51] introduced TI-DBSCAN, which operates directly in the RGB color space and exploits the triangle inequality to significantly reduce neighborhood search complexity while maintaining robustness to noise. A real-time superpixel segmentation framework achieving up to 50 frames per second was presented in [52], employing a two-stage process consisting of fast pixel clustering followed by the merging of small clusters using a robust distance function.

Parameter sensitivity has been addressed through adaptive approaches. Adaptive Parameter DBSCAN [53] estimates ε and minPts automatically based on image size and k-distance analysis, yielding improved segmentation quality and reduced noise. Hybrid strategies combining DBSCAN with other clustering techniques have also been explored. In [54], K-means is used as a pre-processing step for photovoltaic hotspot detection, followed by DBSCAN to identify dense regions of high-temperature pixels. Similarly, integrating Self-Organizing Maps with DBSCAN has been shown to improve segmentation accuracy and computational efficiency [55].

Another effective direction involves superpixel-based pre-processing. Lee [56] applied DBSCAN to SLIC-generated superpixels using an enhanced feature space incorporating CIE Lab color information, compactness, and texture features, resulting in improved region coherence.

Despite these advancements, existing DBSCAN-based segmentation methods primarily rely on density and feature similarity and lack explicit boundary regularization. This limitation motivates the integration of boundary-aware models, such as the Mumford–Shah functional, into the DBSCAN framework to better control cluster expansion across strong image edges while maintaining computational efficiency.

2.4 Mumford-Shah model

Energy minimization formulates image segmentation as an optimization problem in which the objective is to partition an image by minimizing a suitably designed energy functional. This functional penalizes undesirable segmentations, such as noisy or overly complex boundaries, while encouraging desirable properties such as region homogeneity and smooth contours. The Mumford–Shah model, introduced in 1989 [18], is a foundational energy-based formulation widely used for this purpose.

The model seeks a balance between two competing objectives: the segmented regions should closely approximate the original image intensities or colours, and the total length or complexity of the segmentation boundaries should be minimized to avoid over-segmentation and noise [20].

By formalizing these goals, the Mumford–Shah model transforms the qualitative notion of a “good” segmentation into a well-defined mathematical optimization problem.

The Mumford–Shah functional is defined as:

$$E(f, \tau) = \mu^2 \int_{\Omega} \|f - h\|^2 dx dy + \int_{\Omega - \tau} \|\nabla f\|^2 dx dy + \lambda L(\tau) \quad (3)$$

where h is the original image, function f is a piecewise smooth approximation of h and Ω is the image domain partitioned as:

$$\Omega = \bigcup_{i=1}^n \Omega_i \cup \tau$$

Here, τ denotes the set of curves representing region boundaries, μ and λ are positive weighting parameters controlling data fidelity and boundary regularization, respectively, and $L(\tau)$ represents the total boundary length. Larger values of λ encourage fewer and smoother boundaries. The minimization of $E(f, \tau)$ yields the optimal segmentation.

When regions are assumed to be homogeneous, the smoothness term inside regions becomes unnecessary, leading to the simplified piecewise constant Mumford–Shah model:

$$E(f, \tau) = \mu^2 \int_{\Omega} \|f - h\|^2 dx dy + \lambda L(\tau) \quad (4)$$

In this formulation, λ directly controls the smoothness and complexity of the segmentation boundaries.

The Mumford–Shah model and related energy minimization frameworks have been extensively applied to image segmentation, denoising, and restoration tasks [21]. Due to the non-convex and computationally demanding nature of the functional, various numerical optimization techniques have been proposed, including the convex relaxation and splitting-based optimization methods [24] and other implicit numerical methods [25]. Although recent deep learning approaches often replace explicit energy formulations with learned loss functions, the fundamental principle of optimizing a well-defined objective function remains central to modern image segmentation research.

3. COMBINED CLUSTERING AND ENERGY MINIMIZATION APPROACH

Classical k-means clustering applied directly to pixel intensities performs pure color quantization and ignores spatial relationships between pixels, often resulting in fragmented and noisy segments [1], [9]. Because k-means provides no control over spatial continuity or boundary regularity, spatial context must be explicitly incorporated to achieve meaningful image segmentation. Even with spatial constraints, clustering-based methods face difficulties in balancing region connectivity and segment size.

Strong spatial constraints may lead to small, irregular regions, while weaker constraints fail to capture coherent structures. To address these issues, hybrid methods combining efficient clustering with variational energy models have been proposed, notably Mumford–Shah k-means (MS-KM) [23] and the Mumford–Shah Pairwise Nearest Neighbor (MS-PNN) approach [10].

3.1 Mumford–Shah k-means (MS-KM)

MS-KM incorporates a boundary regularization term derived from the Mumford–Shah functional into the k-means objective, allowing pixel assignments to consider both intensity similarity and boundary length [23]. This preserves the computational efficiency of k-means while being substantially faster than classical variational minimization techniques [24], [25]. However, MS-KM inherits the limitations of k-means, including sensitivity to initialization and convergence to local minima.

3.2 Mumford–Shah Pairwise Nearest Neighbor (MS-PNN)

The MS-PNN method adapts an efficient k-nearest neighbor variant of the PNN algorithm to incorporate the Mumford–Shah model while explicitly enforcing spatial connectivity [10]. Experimental results show that MS-PNN outperforms both regular k-means (reg-KM) [50] and MS-KM [23] in segmentation quality, demonstrating the advantage of combining energy-based regularization with agglomerative clustering.

3.3 Mumford–Shah Regularized DBSCAN (Proposed MS-DBSCAN)

The proposed Mumford–Shah Regularized DBSCAN (MS-DBSCAN) integrates density-based clustering with boundary-aware regularization inspired by the Mumford–Shah functional. Classical DBSCAN clusters samples solely based on point density in feature space [32], which often leads to boundary leakage and weak spatial coherence when applied to image data.

To address this limitation, the proposed method embeds spatial proximity, perceptually uniform color similarity, and edge-aware penalties into a unified distance formulation, enabling the segmentation process to respect object boundaries while retaining the ability to discover arbitrarily shaped regions.

Each pixel is represented in a five-dimensional feature space composed of normalized spatial coordinates and CIELAB color components. The LAB color space is selected due to its perceptual uniformity and demonstrated effectiveness in image segmentation tasks [20, 31]. To reduce noise sensitivity and stabilize gradient estimation, a Gaussian smoothing operation is applied prior to clustering, consistent with established variational segmentation practices [18, 25].

3.3.1 Mumford–Shah Regularized Distance

Let a pixel i be represented by the feature vector:

$$x_i = [x_i / W, y_i / H, L_i, a_i, b_i]$$

The Mumford–Shah regularized distance between pixels i and j is defined as:

$$d_{MS}(i, j) = \sqrt{(W_S * [(x_i - x_j)^2 + (y_i - y_j)^2] + W_C * [(L_i - L_j)^2 + (a_i - a_j)^2 + (b_i - b_j)^2]) + \mu * (g_i + g_j)}$$

where W_S and W_C control the relative importance of spatial and color similarity, μ is the boundary regularization parameter, and g_i denotes the local image gradient magnitude. The additive gradient term discourages cluster expansion across strong edges, directly reflecting the boundary-length minimization principle of the Mumford–Shah model [18, 20, 21].

3.3.2 Density-Based Clustering and Region Expansion

Using the distance above, DBSCAN clustering is performed with global parameters ϵ and MinPts. A pixel is classified as a core point if the number of neighbors satisfying:

$$d_{MS}(i, j) \leq \epsilon$$

is greater than or equal to MinPts. Cluster expansion follows the standard DBSCAN breadth-first search strategy [32], but neighborhood queries are governed entirely by the Mumford–Shah regularized distance. This enables robust detection of spatially coherent regions while naturally rejecting noise pixels.

3.3.3 Automatic Region Merging

Although DBSCAN inherently avoids over-segmentation compared to centroid-based clustering, fine-grained partitions may still occur. To address this, an automatic post-processing stage merges adjacent clusters based on mean LAB color similarity. For two spatially adjacent clusters c_1 and c_2 with mean colors μ_{c_1} and μ_{c_2} , merging is performed if:

$$\|\mu_{c_1} - \mu_{c_2}\|_2 < \tau_{\text{merge}}$$

where τ_{merge} is a predefined color similarity threshold.

This step improves region compactness while preserving perceptually meaningful boundaries, analogous to post-regularization in Mumford–Shah-based methods [20, 21].

3.3.4 Comparison with Classical DBSCAN

Table 1 shows the results with different values of parameters like minimum points and EPS. Classical DBSCAN exhibits very high computational cost, increasing sharply with image size and high sensitivity to minimum points. The proposed Mumford–Shah Regularized DBSCAN along with achieving three to four orders of magnitude speedup, produces stable results across different minimum points.




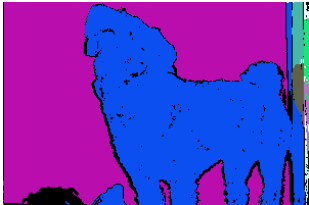



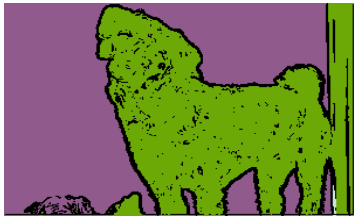


4. CONCLUSION

Classical DBSCAN operates purely on feature density and lacks explicit spatial or boundary awareness. The proposed Mumford–Shah Regularized DBSCAN addresses this limitation by embedding spatial constraints and gradient-based penalties directly into the distance metric. The proposed method offers a practical and scalable alternative for high-quality image segmentation without the need for supervised training or expensive variational optimization.

Future work will focus on extending the method to fully adaptive, locally varying ϵ and MinPts parameters driven by image statistics, enabling improved robustness across heterogeneous regions.

Additional directions include multi-scale processing to better capture fine and coarse structures, and integration with superpixel or graph-based pre-processing to further reduce computational cost. Finally, quantitative evaluation against larger benchmark datasets with pixel-level ground truth will be pursued.

Table 1: Runtime comparison of DBSCAN and Mumford–Shah Regularized DBSCAN

Original			
		Flower	Dog
Image size:		287 x 176	300 x 200
DBSCAN	Minpts = 120 Eps = 18		
	Time in milliseconds:	58571	84692
	Minpts = 50 Eps = 18		
	Time in milliseconds:	33941	69434
MS DBSCAN	MinPts = 20 Eps = 0.12		
	Time in milliseconds:	16	26
	MinPts = 50 Eps = 0.12		
	Time in milliseconds:	14	24

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