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Transient Analysis of Unsymmetrical 2-Phase Induction Machine in Rotor Reference

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Abstract

Analysis of transient performance of unsymmetrical 2-phase induction machine in rotor reference frame is studied. A wide diversity of single phase induction machines can be analyzed with concept of operation of the unsymmetrical 2-phase induction machine. Literature survey revealed that most researches on the transient analysis of single-phase induction machines adopted only the stationary reference frame due to the existence of position dependent variables in other reference frames notably the rotor reference frame. In this paper, mathematical equations that analyze the transient attributes of the 2-phase unsymmetrical machine in the rotor reference frame are established. The model equations are derived to represent single phase capacitor-start induction motor and simulations conducted in MATLAB/Simulink where the results show operational performance of the capacitor-start single phase motor during brief upset. Performance of the simulated model compared and corroborated by that of the stationary reference frame shows a close agreement of the results.

Keywords: *Transient Performance, Capacitor-Start Induction Machine, Rotor Reference Frame, D-q Variables, Unsymmetrical 2-Phase, Simulink Model.*

1. INTRODUCTION

Understanding the behaviors of an induction machine under dynamic conditions such as during startup, abrupt changes in load and the likes is crucial as the changes in speed and load affect the overall performance of the machine. The direct – quadrature (d-q) axis model has been used to investigate the dynamic behavior of induction machines during such conditions [1 – 3]. Dynamic behavior of induction machines studied using different reference frames but mostly with the stationary reference frame by researchers for the unsymmetrical 2-phase induction machine [4, 5]. Concept of operation of the unsymmetrical 2-phase induction machine is relevant to a wide range of single-phase induction machines. The thought describes the dynamic performance of the single-phase machines.

Researchers [6] did analyze transient behaviour of a three-phase induction motor in three reference frames including the stationary reference frame but that is not sufficient for unsymmetrical induction machines since the three-phase induction motor is a symmetrical machine. A concise study of an unsymmetrical induction machine in the stationary reference frame only has been provided by [4]. The work [3] on reference frame theory which involves transforming variables in one reference frame to another has been cited by many researchers. However, the task did not cover reference frame transformation to the rotor reference and synchronously rotating reference frames for unsymmetrical machines. Studies based on the synchronously rotating reference frame for the unsymmetrical induction machine was carried out by [7]. Simulations of the machines in [4] and [7] were done with analog computers which are not very good at handing the non-linear equations that can arise in the analysis of

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unsymmetrical machines. MATLAB/Simulink was however used by [5] to simulate the 2-phase unsymmetrical induction machine in the stationary reference frame. Authors [8] carried out modelling of induction motor on energy based-approach. The work provides a generalization to the port-Hamiltonian model of a squirrel-cage induction motor based on a synchronously rotating reference frame of d-q axes. Furthermore, the port-Hamiltonian formalism is used to provide a modeling approach to a self-excited induction generator which is also deduced for the d-q stationary reference frame. In [9], study of two-phase induction motor with non-orthogonal stator windings in stationary reference frame was presented. The problem studied refers to the determination of the rotor flux of the two-phase induction motor by estimated method and avoid the use of flux sensor.

In this paper, the mathematical equations describing the transient behavior of the unsymmetrical 2-phase induction machine in the rotor reference frame are realized from transformation of the stationary reference. The model of the machine as a capacitor-start single-phase induction motor is simulated in MATLAB/Simulink environment to investigate the machine's performance during startup and load change. The performance is compared with that of unsymmetrical 2-phase induction machine in the stationary reference frame.

2. MODEL OF THE MACHINE

Unsymmetrical 2-phase induction machine can be regarded as a 2-phase machine with nonsymmetrical stator windings and identical rotor windings. Equations describing the performance of the idealized machine can be derived by considering the cross section of an elementary 2-phase induction machine shown in Figure 1. Notations regularly exploited for single-phase induction motors are utilized where possible.

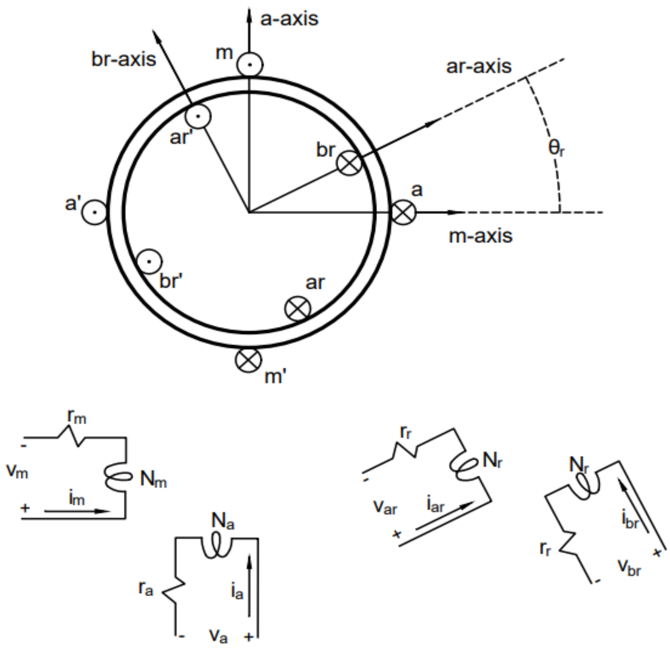

Figure 1: Cross-sectional axes of unsymmetrical 2-phase 2-pole induction machine

The main and auxiliary windings of the stator of the machine are designated *m-m'* and *a-a'* while the rotor windings are designated with *ar-ar'* and *br-br'* respectively. The stator windings are non-identical where the main winding has resistance r_m with N_m effective turns and the auxiliary winding has resistance r_a with N_a effective turns.

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The voltage equations of the machine wingdings can be given in matrix form as:

$$\begin{bmatrix} v_m \\ v_a \end{bmatrix} = \begin{bmatrix} r_m & 0 \\ 0 & r_a \end{bmatrix} \begin{bmatrix} i_m \\ i_a \end{bmatrix} + p \begin{bmatrix} \lambda_m \\ \lambda_a \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} v_{ar} \\ v_{br} \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \end{bmatrix} + p \begin{bmatrix} \lambda_{ar} \\ \lambda_{br} \end{bmatrix} \quad (2)$$

where p = the operator d/dt.

Consider the q-d axes shown in Figure 2 where the rotor moves at speed ω_r and the reference frame moves at speed ω such that at any time interval $\omega_r = p\theta_r$ and $\omega = p\theta$.

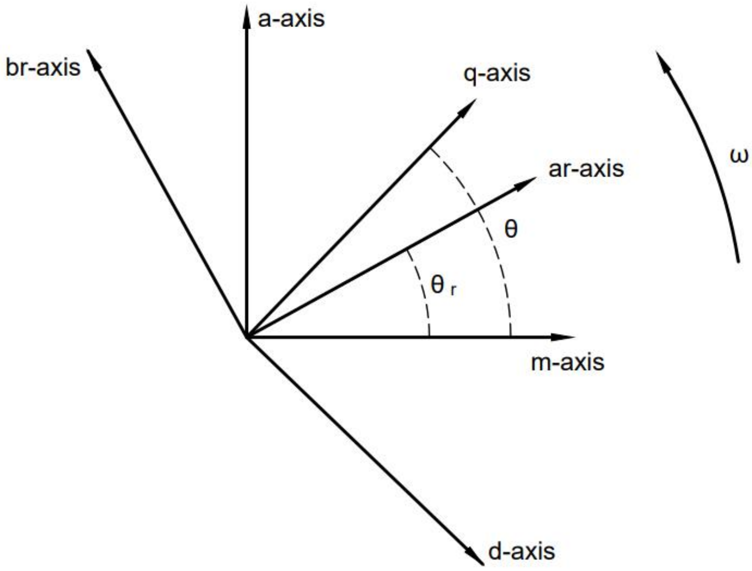

Figure 2: Axes of unsymmetrical 2-phase 2-pole induction machine for an arbitrary reference frame

The stator variables of the machine can be stated as:

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} f_m \\ f_a \end{bmatrix} \quad (3)$$

where f symbolizes voltage, current or flux-linkage variables. Equation (3) in compact form is written as:

$$f_{qds} = K_{\mathbf{2s}} f_{ma} \quad (4)$$

Where

$$K_{2s} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \quad (5)$$

It is most convenient to directly relate variables in one reference frame to another reference frame without involving the main and auxiliary variables in the transformation. Let *x* be the reference frame from which the machine variables are transformed and let *y* denote the reference frame to which the machine variables are being transformed. Then, transformation matrix, ${}^{x}K_{2s}^{y}$, which performs the transformation from reference frame *x* to reference frame *y* is such that

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$$f_{qds}^{y} = {}^{x}K_{2s}^{y} f_{qds}^{x} \qquad (6)$$

Expressing equation (4) in terms of *x* and *y* reference frames gives

$$f_{qds}^{x} = K_{2s}^{x} f_{ma} \qquad (7)$$

$$f_{qds}^{y} = K_{2s}^{y} f_{ma} \qquad (8)$$

Hence,

$$f_{qds}^{y} = {}^{x}K_{2s}^{y} K_{2s}^{x} f_{ma} \qquad (9)$$

So

$$K_{2s}^{y} = {}^{x}K_{2s}^{y} K_{2s}^{x} \qquad (10)$$

$${}^{x}K_{2s}^{y} = K_{2s}^{y} (K_{2s}^{x})^{-1}$$

$$= \begin{bmatrix} \cos\theta_y & \sin\theta_y \\ \sin\theta_y & -\cos\theta_y \end{bmatrix} \begin{bmatrix} \cos\theta_x & \sin\theta_x \\ \sin\theta_x & -\cos\theta_x \end{bmatrix}^{-1} = \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) \end{bmatrix} \qquad (11)$$

Equation (11) is the matrix for transforming stator variables from a reference frame, *x*, to another reference frame, *y*.

In the matrix form, equation (6) is expressed as:

$$\begin{bmatrix} f_{qs}^{y} \\ f_{ds}^{y} \end{bmatrix} = \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) \end{bmatrix} \begin{bmatrix} f_{qs}^{x} \\ f_{ds}^{x} \end{bmatrix} \qquad (12)$$

From Figure 2, the rotor variables of the machine may be expressed as:

$$\begin{bmatrix} f_{qr} \\ f_{dr} \end{bmatrix} = \begin{bmatrix} \cos(\theta - \theta_r) & \sin(\theta - \theta_r) \\ \sin(\theta - \theta_r) & -\cos(\theta - \theta_r) \end{bmatrix} \begin{bmatrix} f_{ar} \\ f_{br} \end{bmatrix} \qquad (13)$$

In compact form, equation (13) becomes

$$f_{qdr} = K_{2r} f_{abr} \qquad (14)$$

where

$$K_{2r} = \begin{bmatrix} \cos(\theta - \theta_r) & \sin(\theta - \theta_r) \\ \sin(\theta - \theta_r) & -\cos(\theta - \theta_r) \end{bmatrix} \qquad (15)$$

In the same manner of equations (6) - (11) for the stator variables, the reference frame transformation matrix for rotor variables can be expressed as:

$${}^{x}K_{2r}^{y} = K_{2r}^{y} (K_{2r}^{x})^{-1} \qquad (16)$$

where

$$K_{2r}^{y} = \begin{bmatrix} \cos(\theta_y - \theta_r) & \sin(\theta_y - \theta_r) \\ \sin(\theta_y - \theta_r) & -\cos(\theta_y - \theta_r) \end{bmatrix} \qquad (17)$$

$$K_{2r}^{x} = \begin{bmatrix} \cos(\theta_x - \theta_r) & \sin(\theta_x - \theta_r) \\ \sin(\theta_x - \theta_r) & -\cos(\theta_x - \theta_r) \end{bmatrix} \qquad (18)$$

Hence,

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$$^{x}K_{2r}^{y} = \begin{bmatrix} \cos(\theta_y - \theta_r) & \sin(\theta_y - \theta_r) \\ \sin(\theta_y - \theta_r) & -\cos(\theta_y - \theta_r) \end{bmatrix} \begin{bmatrix} \cos(\theta_x - \theta_r) & \sin(\theta_x - \theta_r) \\ \sin(\theta_x - \theta_r) & -\cos(\theta_x - \theta_r) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) \end{bmatrix} \quad (19)$$

This shows that $^{x}K_{2r}^{y} = {}^{x}K_{2s}^{y}$. Therefore,

$$\begin{bmatrix} f_{qr}^{y} \\ f_{dr}^{y} \end{bmatrix} = \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) \end{bmatrix} \begin{bmatrix} f_{qr}^{x} \\ f_{dr}^{x} \end{bmatrix} \quad (20)$$

The voltage equations of a 2-phase unsymmetrical induction machine is given in the stationary reference frame [4] as:

$$\begin{bmatrix} v_{qs}^{s} \\ v_{ds}^{s} \end{bmatrix} = \begin{bmatrix} r_m & 0 \\ 0 & r_a \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs}^{s} \\ \lambda_{ds}^{s} \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} v_{qr}^{s} \\ v_{dr}^{s} \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix} \begin{bmatrix} i_{qr}^{s} \\ i_{dr}^{s} \end{bmatrix} - \omega \begin{bmatrix} \lambda_{dr}^{s} \\ -\lambda_{qr}^{s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qr}^{s} \\ \lambda_{dr}^{s} \end{bmatrix} \quad (22)$$

where

$$\begin{bmatrix} \lambda_{qs}^{s} \\ \lambda_{ds}^{s} \end{bmatrix} = \begin{bmatrix} L_m & 0 \\ 0 & L_a \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \end{bmatrix} + \begin{bmatrix} M_{mr} & 0 \\ 0 & M_{ar} \end{bmatrix} \begin{bmatrix} i_{qr}^{s} \\ i_{dr}^{s} \end{bmatrix} \quad (23)$$

$$\begin{bmatrix} \lambda_{qr}^{s} \\ \lambda_{dr}^{s} \end{bmatrix} = \begin{bmatrix} L_r & 0 \\ 0 & L_r \end{bmatrix} \begin{bmatrix} i_{qr}^{s} \\ i_{dr}^{s} \end{bmatrix} + \begin{bmatrix} M_{mr} & 0 \\ 0 & M_{ar} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \end{bmatrix} \quad (24)$$

$$L_m = L_{lm} + L_{mm} \quad (25)$$

$$L_a = L_{la} + L_{ma} \quad (26)$$

$$L_r = L_{lr} + L_{mr} \quad (27)$$

L_{lm}, L_{la}, L_{lr} are leakage inductances in main, auxiliary and rotor windings correspondingly.

L_{mm}, L_{ma}, L_{mr} are magnetizing inductances in main, auxiliary and rotor windings correspondingly.

M_{mr} and M_{ar} are amplitudes of the mutual inductances between main and rotor windings, and auxiliary and rotor windings correspondingly.

Putting equations (25) – (27) into equations (23) and (24), and knowing that the maximum mutual inductance between any two coils cannot exceed the magnetizing inductance in any of the coils gives

$$\begin{bmatrix} \lambda_{qs}^{s} \\ \lambda_{ds}^{s} \end{bmatrix} = \begin{bmatrix} L_{lm} & 0 \\ 0 & L_{la} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \end{bmatrix} + \begin{bmatrix} M_{mr} & 0 \\ 0 & M_{ar} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} + i_{qr}^{s} \\ i_{ds}^{s} + i_{dr}^{s} \end{bmatrix} \quad (28)$$

$$\begin{bmatrix} \lambda_{qr}^{s} \\ \lambda_{dr}^{s} \end{bmatrix} = \begin{bmatrix} L_{lr} & 0 \\ 0 & L_{lr} \end{bmatrix} \begin{bmatrix} i_{qr}^{s} \\ i_{dr}^{s} \end{bmatrix} + \begin{bmatrix} M_{mr} & 0 \\ 0 & M_{ar} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} + i_{qr}^{s} \\ i_{ds}^{s} + i_{dr}^{s} \end{bmatrix} \quad (29)$$

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The voltage equations of the machine in the stationary reference frame now becomes

$$\begin{bmatrix} v_{qs}^{s} \\ v'^{s}_{ds} \end{bmatrix} = \begin{bmatrix} r_m & 0 \\ 0 & r'_a \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i'^{s}_{ds} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs}^{s} \\ \lambda'^{s}_{ds} \end{bmatrix} \quad (30)$$

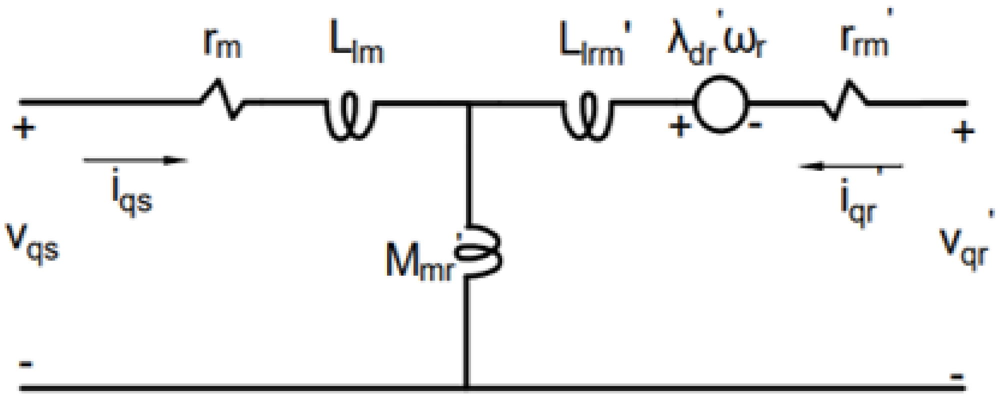
$$\begin{bmatrix} v_{qr}^{s} \\ v'^{s}_{dr} \end{bmatrix} = \begin{bmatrix} r'_r & 0 \\ 0 & r'_r \end{bmatrix} \begin{bmatrix} i'^{s}_{qr} \\ i'^{s}_{dr} \end{bmatrix} - \omega \begin{bmatrix} \lambda'^{s}_{dr} \\ -\lambda'^{s}_{qr} \end{bmatrix} + p \begin{bmatrix} \lambda'^{s}_{qr} \\ \lambda'^{s}_{dr} \end{bmatrix} \quad (31)$$

where

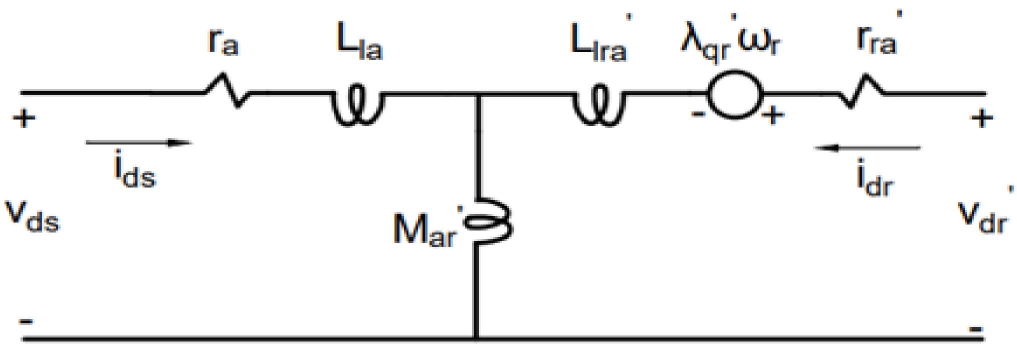
$$\begin{bmatrix} \lambda_{qs}^{s} \\ \lambda'^{s}_{ds} \end{bmatrix} = \begin{bmatrix} L_{lm} & 0 \\ 0 & L'_{la} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \end{bmatrix} + \begin{bmatrix} M'_{mr} & 0 \\ 0 & M'_{ar} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} + i'^{s}_{qr} \\ i'^{s}_{ds} + i'^{s}_{dr} \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} \lambda_{qr}^{s} \\ \lambda_{dr}^{s} \end{bmatrix} = \begin{bmatrix} L_{lr} & 0 \\ 0 & L_{lr} \end{bmatrix} \begin{bmatrix} i_{qr}^{s} \\ i_{dr}^{s} \end{bmatrix} + \begin{bmatrix} M'_{mr} & 0 \\ 0 & M'_{ar} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} + i'^{s}_{qr} \\ i'^{s}_{ds} + i'^{s}_{dr} \end{bmatrix} \quad (33)$$

In this development of the unsymmetrical 2-phase induction machine, the q quantities will be referred the m winding and the d quantities will be referred to the a winding. The developed equivalent circuits the machine are shown in Figures 3(a) and (b).



(a)



(b)

Figure 3: d – q axes equivalent circuits of unsymmetrical 2-phase induction machine

Referring all d-axis and rotor variables to the main winding of the stator gives the variables in Table 1.

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Table 1: Machine variables

Referred variables	Actual variables	Referred variables	Actual variables
v'_{ds}	$\frac{N_m}{N_a} v^s_{ds}$	λ'_{qr}	$\frac{N_m}{N_a} \lambda^s_{qr}$
v'_{qr}	$\frac{N_m}{N_r} v^s_{qr}$	λ'_{dr}	$\frac{N_m}{N_a} \lambda^s_{dr}$
v'_{dr}	$\frac{N_m}{N_r} v^s_{dr}$	L'_{la}	$\left(\frac{N_m}{N_a}\right)^2 L_{la}$
i'_{ds}	$\frac{N_m}{N_a} i^s_{ds}$	M'_{mr}	$\frac{N_m}{N_r} M_{mr}$
i'_{qr}	$\frac{N_m}{N_r} i^s_{qr}$	M'_{ar}	$\frac{N_m}{N_r} M_{ar}$
i'_{dr}	$\frac{N_m}{N_r} i^s_{dr}$	r'_a	$\left(\frac{N_m}{N_a}\right)^2 r_a$
λ'_{ds}	$\frac{N_m}{N_a} \lambda^s_{ds}$	r'_r	$\left(\frac{N_m}{N_a}\right)^2 r_r$

A. Rotor Reference Frame

Transforming voltage, current and flux-linkage variables from the stationary reference frame to the rotor reference frame, then,

$$\begin{bmatrix} v^r_{qs} \\ v^r_{ds} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} v^s_{qs} \\ v^s_{ds} \end{bmatrix} \tag{34}$$

$$\begin{bmatrix} v^r_{qr} \\ v^r_{dr} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} v^s_{qr} \\ v^s_{dr} \end{bmatrix} \tag{35}$$

$$\begin{bmatrix} i^r_{qs} \\ i^r_{ds} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i^s_{qs} \\ i^s_{ds} \end{bmatrix} \tag{36}$$

$$\begin{bmatrix} i^r_{qr} \\ i^r_{dr} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i^s_{qr} \\ i^s_{dr} \end{bmatrix} \tag{37}$$

$$\begin{bmatrix} \lambda^r_{qs} \\ \lambda^r_{ds} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \lambda^s_{qs} \\ \lambda^s_{ds} \end{bmatrix} \tag{38}$$

$$\begin{bmatrix} \lambda^r_{qr} \\ \lambda^r_{dr} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \lambda^s_{qr} \\ \lambda^s_{dr} \end{bmatrix} \tag{39}$$

Substituting equations (21) and (22) into equations (34) and (35) respectively and further transforming the current and flux-linkage variables from the resulting equation into rotor reference frame variables gives the voltage equations of a 2-phase unsymmetrical induction machine in the rotor reference frame as:

$$\begin{bmatrix} v^r_{qs} \\ v^r_{ds} \end{bmatrix} = \frac{1}{2}(r_m + r_a)\begin{bmatrix} i^r_{qs} \\ i^r_{ds} \end{bmatrix} + \frac{1}{2}(r_m - r_a)\begin{bmatrix} cos2\theta_r & sin2\theta_r \\ sin2\theta_r & -cos2\theta_r \end{bmatrix}\begin{bmatrix} i^r_{qs} \\ i^r_{ds} \end{bmatrix} + \omega_r \begin{bmatrix} \lambda^r_{ds} \\ -\lambda^r_{qs} \end{bmatrix} + p\begin{bmatrix} \lambda^r_{qs} \\ \lambda^r_{ds} \end{bmatrix} \tag{40}$$

$$\begin{bmatrix} v^r_{qr} \\ v^r_{dr} \end{bmatrix} = r_r \begin{bmatrix} i^r_{qr} \\ i^r_{dr} \end{bmatrix} + p\begin{bmatrix} \lambda^r_{qr} \\ \lambda^r_{dr} \end{bmatrix} \tag{41}$$

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Referring all quantities to the main winding,

$$\begin{bmatrix} v_{qs}^{r} \\ v_{ds}^{\prime r} \end{bmatrix} = r_{\alpha s}\begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{\prime r} \end{bmatrix} + r_{\beta s}\begin{bmatrix} cos2\theta_r & sin2\theta_r \\ sin2\theta_r & -cos2\theta_r \end{bmatrix}\begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{\prime r} \end{bmatrix} + \omega_r\begin{bmatrix} \lambda_{ds}^{\prime r} \\ -\lambda_{qs}^{r} \end{bmatrix} + p\begin{bmatrix} \lambda_{qs}^{r} \\ \lambda_{ds}^{\prime r} \end{bmatrix} \quad (42)$$

$$\begin{bmatrix} v_{qr}^{\prime r} \\ v_{dr}^{\prime r} \end{bmatrix} = r_r^{\prime}\begin{bmatrix} i_{qr}^{\prime r} \\ i_{dr}^{\prime r} \end{bmatrix} + p\begin{bmatrix} \lambda_{qr}^{\prime r} \\ \lambda_{dr}^{\prime r} \end{bmatrix} \quad (43)$$

where

$$r_{\alpha s} = \frac{1}{2}(r_m + r_a^{\prime}) \quad (44)$$

$$r_{\beta s} = \frac{1}{2}(r_m - r_a^{\prime}) \quad (45)$$

B. Torque

The totality of the instantaneous power input to all stator and rotor windings can be conveyed in the rotor reference frame as:

$$p_{in} = v_{qs}^{r}i_{qs}^{r} + v_{ds}^{\prime r}i_{ds}^{\prime r} + v_{qr}^{\prime r}i_{qr}^{\prime r} + v_{dr}^{\prime r}i_{dr}^{\prime r}$$

$$= (r_{\alpha s} + r_{\beta s}\cos 2\theta r){i_{qs}^{r}}^{2} + r_{\beta s}\, i_{qs}^{r}i_{dr}^{\prime r}\sin 2\theta_r + \omega_r i_{qs}^{r}\lambda_{ds}^{\prime r} + i_{qs}^{r}\left(p\lambda_{qs}^{r}\right) + (r_{\alpha s} - r_{\beta s}\cos 2\theta r){i_{ds}^{\prime r}}^{2}$$

$$+ r_{\beta s}\, i_{qs}^{r}i_{dr}^{\prime r}\sin 2\theta_r - \omega_r i_{ds}^{\prime r}\lambda_{qs}^{r} + i_{ds}^{\prime r}(p\lambda_{ds}^{\prime r}) + r_r^{\prime}{i_{qr}^{\prime r}}^{2} + i_{qr}^{\prime r}(p\lambda_{qr}^{\prime r}) + r_r^{\prime}{i_{dr}^{\prime r}}^{2} + i_{dr}^{\prime r}(p\lambda_{dr}^{\prime r}) \quad (46)$$

Eliminating terms associated with copper loss and derivatives gives the mechanical power input to the shaft expressed as:

$$p_{mech} = (\lambda_{ds}^{\prime r}i_{qs}^{r} - \lambda_{qs}^{r}i_{ds}^{\prime r})\omega_r \quad (47)$$

The mechanical power can also be expressed in terms of electromechanical torque developed such that $p_{mech} = T_{em}^{r}\omega_{rm}$, where ω_{rm} is the mechanical speed expressed as $(2/P)\omega_r$ [5] and P is the pole number. The torque developed is then expressed in the rotor reference frame as:

$$T_{em}^{r} = \frac{P}{2}(\lambda_{ds}^{\prime r}i_{qs}^{r} - \lambda_{qs}^{r}i_{ds}^{\prime r}) \quad (48)$$

The electromagnetic torque in the stationary reference frame can also be realized in the same manner. The input power is

$$p_{in}^{s} = v_{qs}^{s}i_{qs}^{s} + v_{ds}^{\prime s}i_{ds}^{\prime s} + v_{qr}^{\prime s}i_{qr}^{\prime s} + v_{dr}^{\prime s}i_{dr}^{\prime s}$$

$$= (r_m {i_{qs}^{s}}^{2} + i_{qs}^{s}p\lambda_{qs}) + (r_a^{\prime}{i_{ds}^{\prime s}}^{2} + i_{ds}^{\prime s}p\lambda_{ds}^{\prime s}) + \left(r_r^{\prime}{i_{qr}^{\prime s}}^{2} - i_{qr}^{\prime s}\lambda_{dr}^{\prime s}p\theta_r + i_{qr}^{\prime s}p\lambda_{qr}^{\prime s}\right)$$

$$+ (r_r^{\prime}{i_{qr}^{\prime s}}^{2} - i_{qr}^{\prime s}\lambda_{dr}^{\prime s}p\theta_r + i_{qr}^{\prime s}p\lambda_{qr}^{\prime s}) \quad (49)$$

Eliminating terms associated with copper loss and derivatives gives the power and torque as:

$$p_{mech}^{s} = (\lambda_{qr}^{\prime s}i_{dr}^{\prime s} - \lambda_{dr}^{\prime s}i_{qr}^{\prime s})\omega_r \quad (50)$$

$$T_{em}^{s} = \frac{P}{2}(\lambda_{qr}^{\prime s}i_{dr}^{\prime s} - \lambda_{dr}^{\prime s}i_{qr}^{\prime s}) \quad (51)$$

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3. SIMULATION OF THE MACHINE

The parameters of the capacitor-start single-phase induction machine used for the simulation are indicated in Table 2. The machine is a ¼-hp, 110-volt, 4-pole capacitor-start induction motor.

Table 2: Specifications of the capacitor-start induction motor [4]

Specifications	Ratings	Specifications	Ratings
HP Rating (VA)	186.5	Auxiliary winding resistance (Ω)	7.14
Rated r. m. s. voltage (V)	110	Auxiliary leakage reactance (Ω)	3.22
Number of poles	4	Rotor leakage reactance referred to the main winding (Ω)	2.12
Rated frequency (Hz)	60	Rotor resistance referred to the main winding (Ω)	4.12
Main winding to auxiliary winding turns ratio	1/1.18	Magnetizing reactance referred to the main winding (Ω)	66.8
Main winding resistance (Ω)	2.02	Rotor inertia (kg.m^2)	0.0146
Main leakage reactance (Ω)	2.79		

To simulate the single-phase induction machine using Simulink, two subsystems are modeled: the electrical subsystem (which handle voltage, current and flux per second variables) and the mechanical subsystem handles mechanical quantities including torque, speed and slip. While Figure 4 gives the Simulink model of the electrical subsystem, Figure 5 gives that of the mechanical subsystem of the single-phase induction motor.

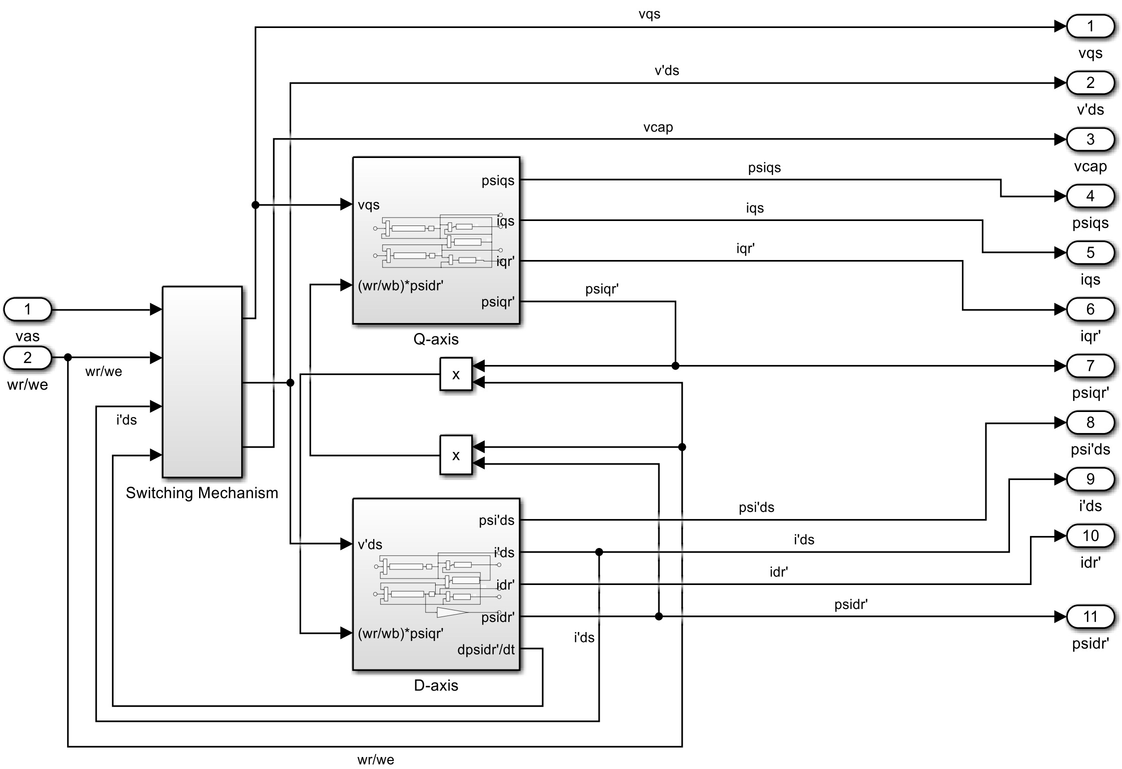

Figure 4: Model of electrical subsystem of the machine

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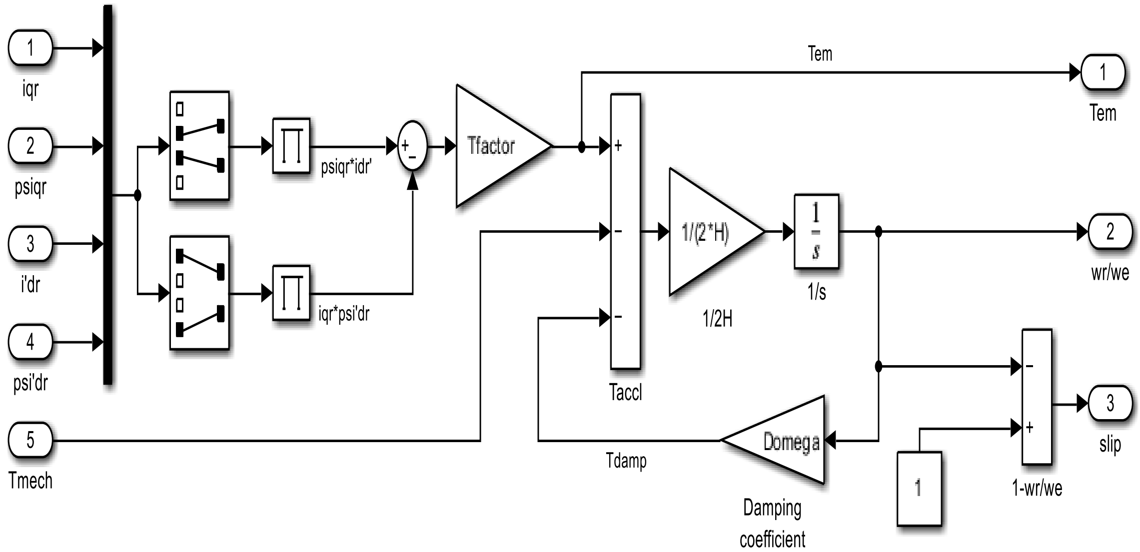

Figure 5: Model of mechanical subsystem of the machine

The transformations conveyed in equations (12) and (20) are modeled as shown in Figure 6. The output of this subsystem depends on the rotor position.

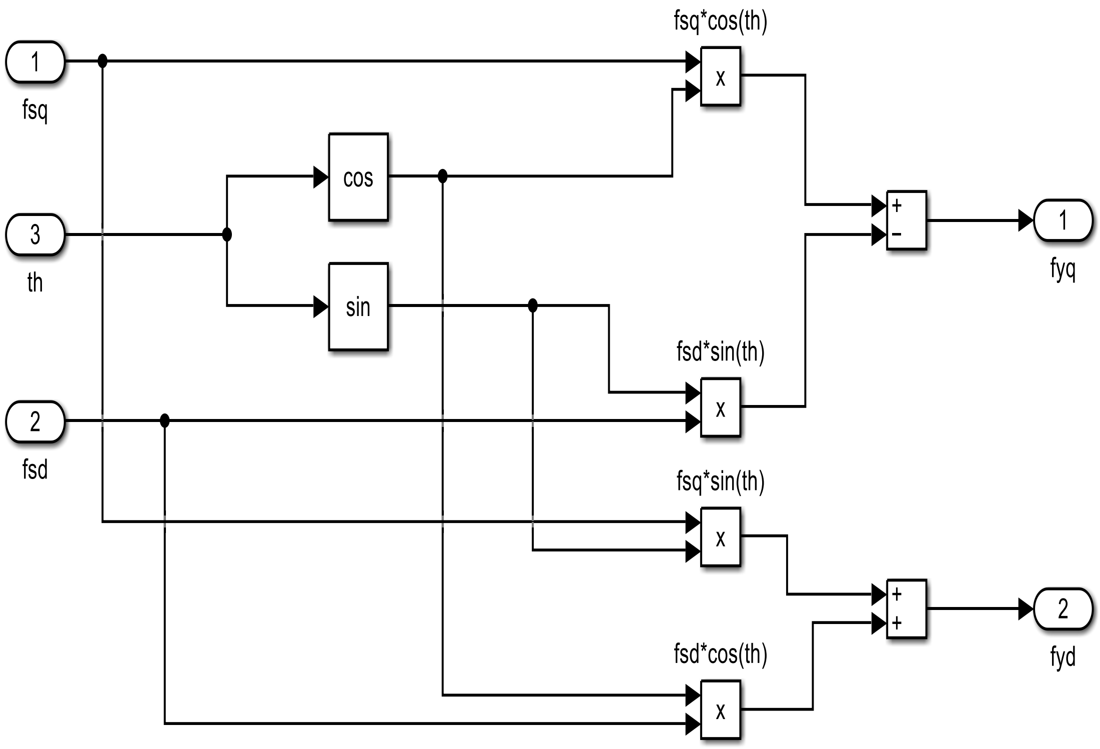

Figure 6: Reference frame transformation for both rotor and stator variables

Figure 7 shows the complete simulink model for the single-phase induction machine in the rotor reference frame. This model has embedded the Power measurement subsystem whose output is a vector.

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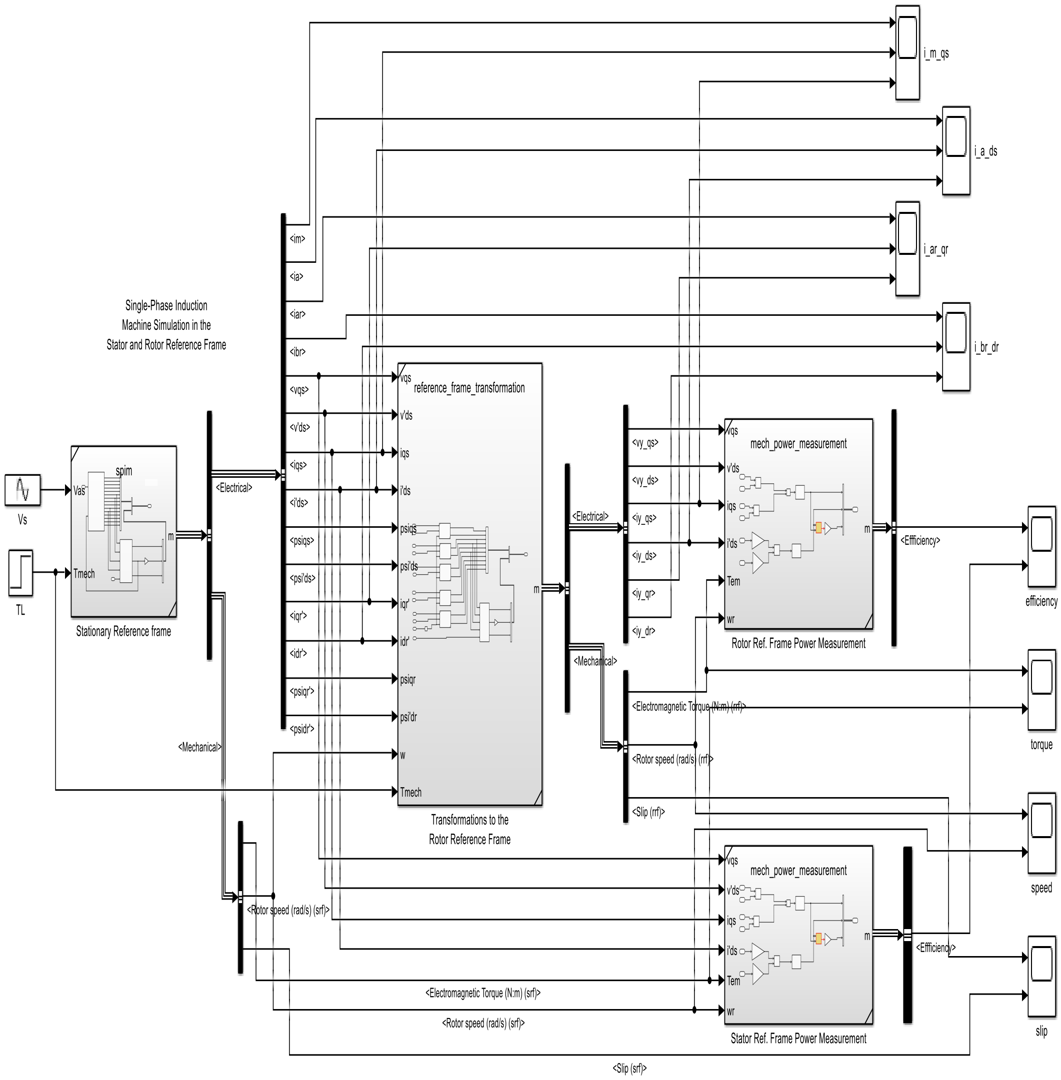

Figure 7: Single-phase induction machine simulation in the rotor reference frame

4. RESULTS AND DISCUSSION

The simulation was run using Simulink models of the machine for a computer time 4 seconds. While starting the machine at no-load, the rated torque of 0.98941N.m was then applied to the motor after 2 seconds. Simulation results of the machine in the two reference frames were observed with Simulink scopes and compared.

Figure 8 shows the rotor speed attributes of machine in both stationary and rotor reference frames. The speed in the rotor reference frame averagely is slightly lower than that in the stationary reference frame.

This was observed at no-load and the machine was loaded. The rotor reference frame speed is 188.2 rad/s which is almost the same with the synchronous speed of the motor of 188.5 rad/s.

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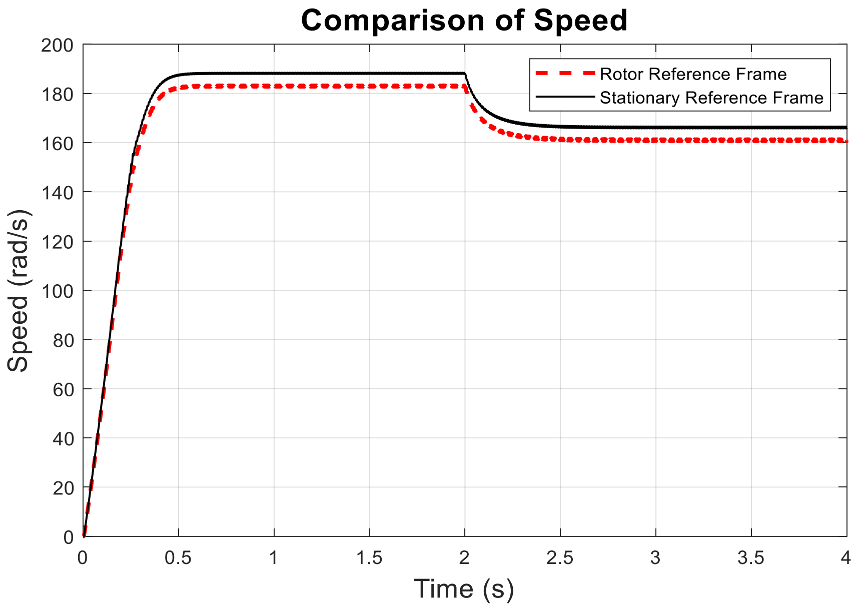

Figure 8: Rotor speed characteristics

Figure 9 indicates the stator main winding current of the machine behaving exactly as the q-axis stator current in the stationary reference frame. The result also shows a clear difference between the main winding current and the q-axis stator current in the rotor reference frame.

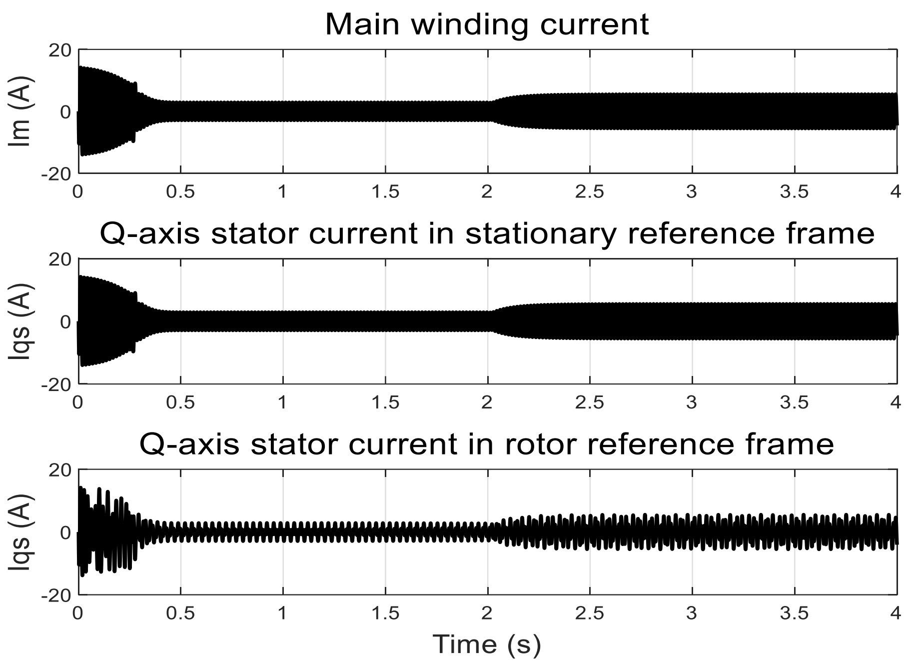

Figure 9: Main winding and stator q-axis currents

In the same manner, Figure 10 indicates the d-axis stator current of the rotor reference frame being completely different from the current in the auxiliary winding. Comparison of the current in the stationary reference frame to the auxiliary winding current shows that they are exactly the same except that they are mirror images of one another. This can be justified by

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recalling that the choice of the direction of the d-axis was made to be directly opposite the a-axis whenever the q-axis coincides with the m-axis as depicted in Figure 2.

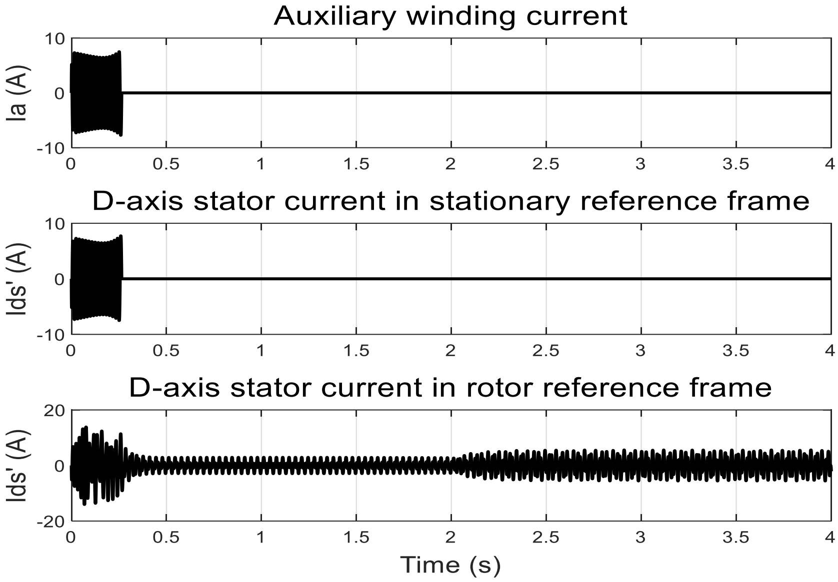

Figure 10: Auxiliary and stator d-axis currents

Comparing the rotor q-axis current of the rotor reference frame to the actual current in the rotor windings of the motor shows that they are identical as revealed in Figure 11. This is a justification of the fact that the rotor reference frame moves at the very relative speed as the rotor. Compared to the q-axis rotor current in the stationary reference frame, a clear difference is seen.

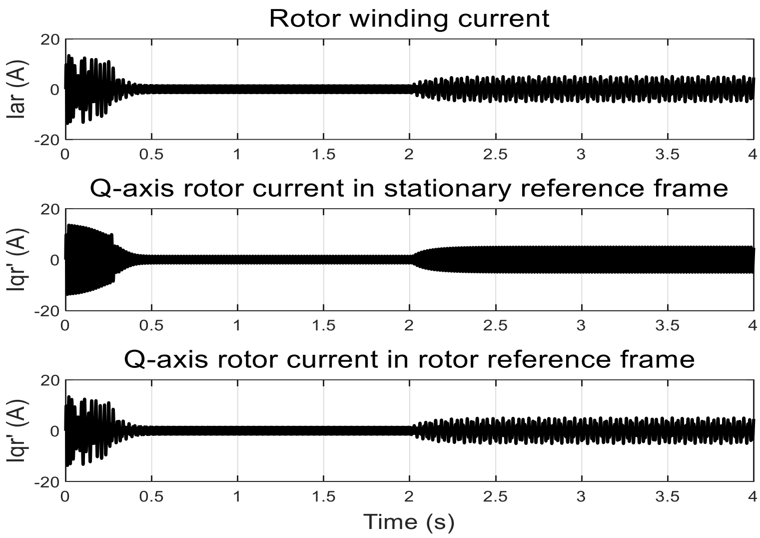

Figure 11: q-axis rotor currents

The behavior is also observed between the actual rotor winding currents and the d-axis currents in both reference frames revealed in Figure 12. The rotor reference frame is a mirror image of the physical rotor current whereas the stationary reference frame current behaves completely different. In fact, comparing both the q-axis and d-axis rotor currents in the stationary reference frame shows that the two quantities are not the same. This contrasts the

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assumption that the rotor windings are identical – an assumption which on the other hand, holds true for the rotor reference frame.

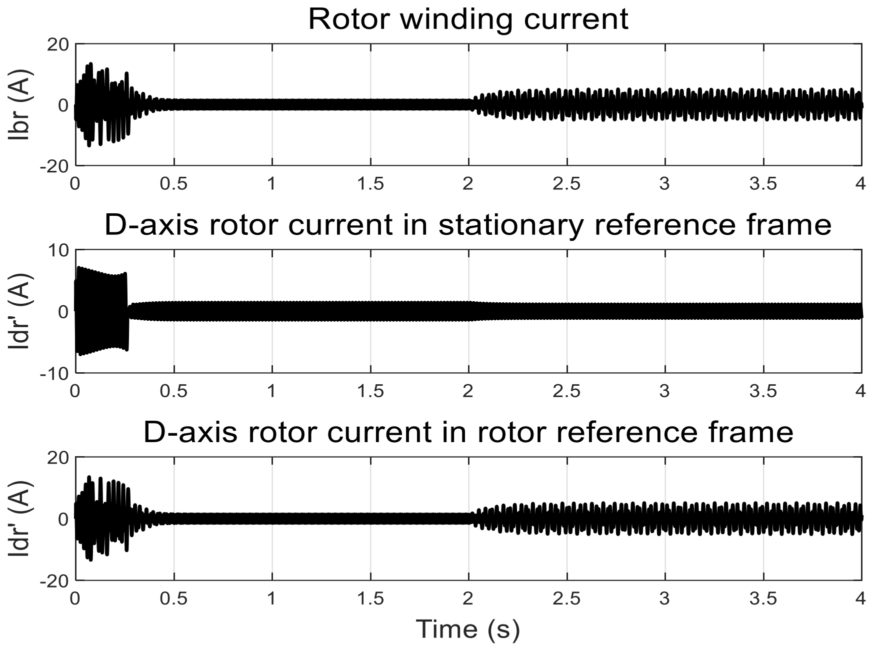

Figure 12: d-axis rotor currents

Electromagnetic torque of the machine behaves the same in both reference frames as seen in Figure 13. This can be justified on account of the fact that electromagnetic torque is produced by interplay between rotor currents and the stator's magnetic field. Therefore, the torque is the same regardless of the reference frame, albeit different equations of electromagnetic torque have been derived for each reference frame.

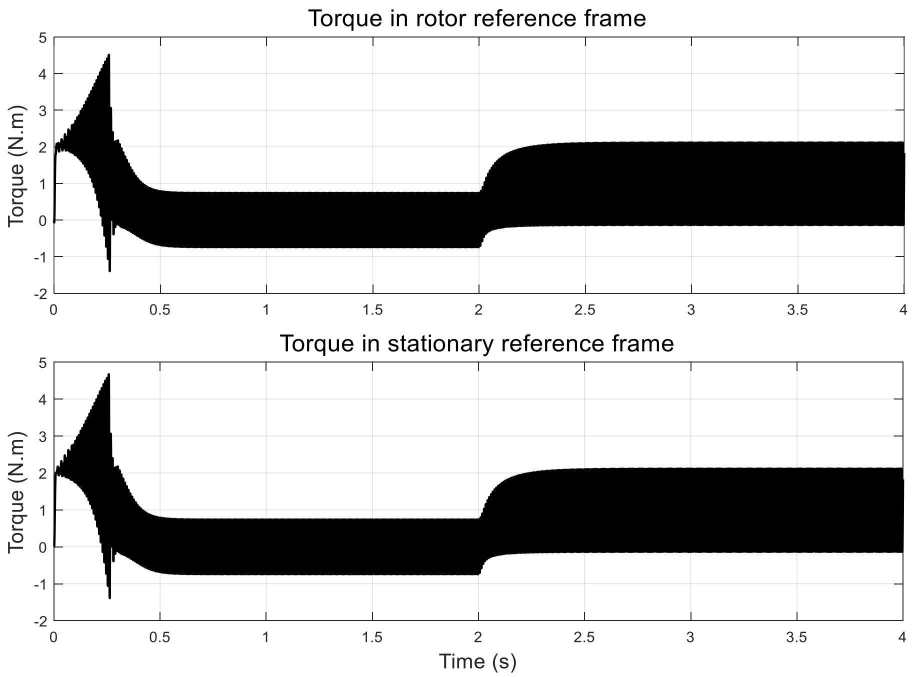

Figure 13: Electromagnetic torque

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The idealized machine in rotor reference frame from the stationary reference frame shows a slightly lower efficiency as indicated in Figure 14.

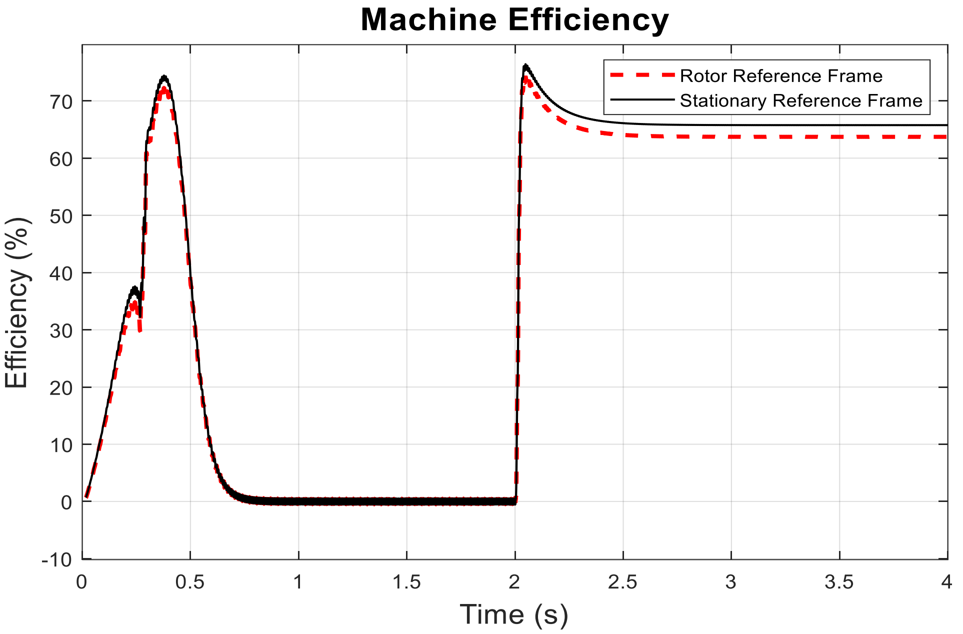

Figure 14: Efficiency of the machine

For the stationary reference frame, the efficiency at rated torque gracefully settles to a steady value of 65.78% from 0% at no-load. For the rotor reference frame, the plot pattern is very similar to the stationary frame but the efficiency settles at 63.72%. This indicates that, while the motor can handle the rated torque, it settles into a slightly lower efficiency due to internal losses stabilizing at this load. The slight difference in efficiency peak and steady-state values compared to the stationary frame reflects the rotor reference frame accounting for more rotor-related losses.

5. CONCLUSION

This paper investigates the transient behavior of an unsymmetrical 2-phase induction machine in the rotor reference frame. Mathematical equations describing the transient behavior of a 2-phase unsymmetrical machine in both the stationary reference frame and the rotor reference frame are derived. MATLAB Simulink is used to model a capacitor-start single-phase induction motor from the model equations set forth. The results indicate the transient performance attributes of the machine in the two reference frames. Critical observation of the results can be surmised that: for the stationary reference frame, there is no difference between the currents i_m and i_{qs}^s. The auxiliary winding current, i_a and $i_{dr}^{\prime s}$ are also the same except that they are mirror images of one another. In addition, i_{ar} and $i_{qr}^{\prime r}$ are identical for the rotor reference frame. The d-axis current, $i_{dr}^{\prime r}$ is a mirror image of i_{br}. As a result of the identical rotor windings, i_{ar} is the same as i_{br} while $i_{dr}^{\prime r}$ is identical to $i_{qr}^{\prime r}$. The electromagnetic torque developed is the same regardless of the reference frame used.

Furthermore, the study shows that a single-phase induction machine is slightly less efficient in the rotor reference frame compared to when analyzed using the stationary reference frame. In short, it can be seen that the rotor reference frame is best suited for machine transient analysis involving rotor variables such as rotor current, speed and torque. The lower efficiency

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of the motor in the rotor reference frame indicates that the rotor reference frame is less idealized than the stationary reference frame but reports more rotor-related losses. The representations that have been developed are instructional enabling the demonstration of the complete dynamic characteristics of single-phase induction machine.

Conflict of Interest

The authors declare no conflicts of interests.

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