

On the Thermal Behavior Response of Laminated Composite Plates via FEM

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Abstract

This research work was devoted to finite element numerical modeling of the thermal behavior in temperature variation of composite material plates. This plate is made up of three layers of a symmetrical laminate and subjected to thermal loads in the form of a thermal gradient. The aim is to highlight the different states of thermal stress responsible for damage to the structure. We adopt quadrangle finite elements at four nodes with two degrees of freedom per node, the results obtained in the form of a nodal solution show in great detail the stress most influenced on composite plates among three different stresses. In this numerical modeling, we try to highlight the problem of variation in the temperature of the composite material plate, and to visualize the results to be able to derive the optimal Young's modulus ratio and the best orientation of the fibers under stress. Thermal load. The types and architectures of laminate were the subject of a detailed analysis. To predict the thermal behavior of composite plates and the influences of type and architectures of laminates on thermal stresses, numerical analysis by EF was performed and the results are expressed as graphs. The scenario analyzed is the stress field in Cartesian coordinates as a function of the fiber orientation angle, thermal load and the orthotropic ratio. The parameters that influence thermal behavior of thermal gradient materials are: time, applied thermal load, laminate type and material architecture. Thermal variation is a key factor that controls thermal behaviors. Finite element numerical modeling also shows the state of thermal behavior of the laminated plates, characterized by the maximum thermal stress. The results of this modeling also make it possible to define a cold room manufacturing strategy.

Keywords: *Numerical Modeling, Composite Plate, Thermal Stresses, Finite Elements.*

1. INTRODUCTION

Numerical modeling using the finite element method consists on the one hand of determining the displacement of the points of the structure under various stresses, and on the other hand of properly formulating the variation of the energy, which makes it possible to deduce the stresses and deformations. Thus, researchers studied the choice of displacement field, as well as the formulation of energy variation [1, 2].

The analysis of the dynamic behavior of structures made of composite materials was recently developed in a synthesis work developed by Berthelot [3].

A numerical study using the element method of free vibrations of laminated plates was studied in [4]. The author also derived a set of variational equilibrium equations consisting of the kinematic models initially proposed by Levinson and Murthy [5].

In [6], finite element and analytical solutions to predict the behavior of laminated composite plates. Using finite element solutions for free vibration analysis of laminated composite plates were also obtained in [7].

The study of the mechanical behavior of laminates has, until now, been carried out considering that the material was related to a temperature reference state, for which the deformation field and the stress field in the material were considered to be zero in the absence of mechanical loading.

In practice, structures are subject to temperature variations both during their implementation and during their use. The first effect of temperature variation is to modify the rigidity and breaking characteristics of the material. Additionally, temperature variation produces thermal expansion of the material.

The distribution of temperatures in the structure and over time is determined from heat transfer phenomena. In practice, thermal and swelling phenomena only produce extensions or contractions, not affecting shear deformations.

The most interesting feature of this analysis is that it allows finite elements of quadrilateral shape at 4 knots and 2 ddl/nodes of the plates in symmetric laminated composites of different fiber orientations.

2. NUMERICAL MODELING BY THE FINITE ELEMENT METHOD

Consider a square plate ($L \times L \times h$) having three orthotropic symmetrical layers with the coordinate system shown in Figure 1.

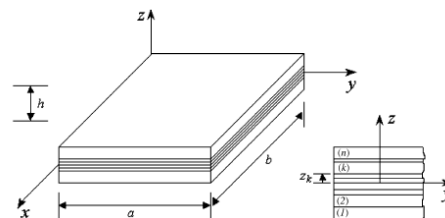


Figure 1: Coordinate system and layer numbering used for a laminate plate type.

2.1. Iso-parametric representation

The iso-parametric formulations are used for the task from the numerical calculation point of view. Thus, all approximations made on the real element will be replaced by approximations on the reference element, the derivatives of the Cartesian coordinates x, y replaced by the derivatives of the iso-parametric coordinates ξ, η . The coordinates $x(\xi, \eta)$ and $y(\xi, \eta)$ of any point (ξ, η) are defined by:

$$\begin{cases} x(\xi, \eta) = \sum_{i=1}^4 N_i x_i \\ y(\xi, \eta) = \sum_{i=1}^4 N_i y_i \end{cases} \quad (1)$$

Figure 2 depicts the mesh of the laminate plate according to the four-nodes rectangular element with 2 DOF

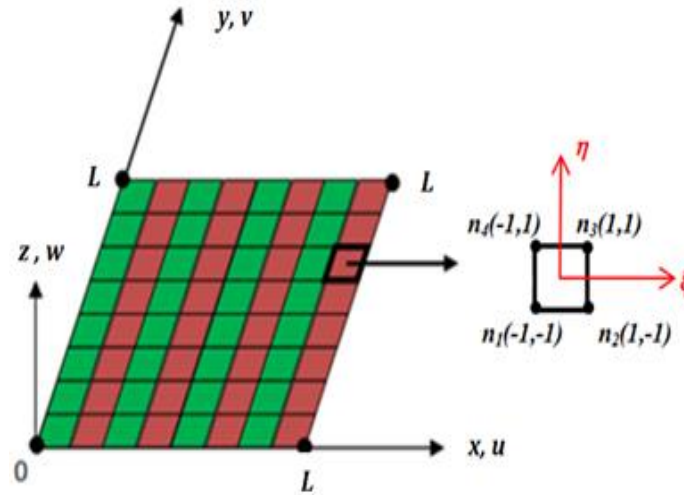


Figure 2: Laminated plate mesh.

Where (x_i, y_i) are the coordinates of node i , and the quadratic interpolation functions are given by:

$$N = [N_1 \quad N_2 \quad N_3 \quad N_4] \tag{2}$$

In the case where the structure is meshed according to laminated elements with four nodes (figure 2), the field of membrane displacements at a point is expressed by interpolation as a function of these degrees of freedom at the following four nodes:

$$\begin{cases} u^e(x, y) = N_1(x, y)u_1(x, y) + N_2(x, y)u_2(x, y) + N_3(x, y)u_3(x, y) + N_4(x, y)u_4(x, y) \\ v^e(x, y) = N_1(x, y)v_1(x, y) + N_2(x, y)v_2(x, y) + N_3(x, y)v_3(x, y) + N_4(x, y)v_4(x, y) \end{cases} \tag{3}$$

Where u_i and v_i with $i = 1, 2, 3, 4$ are the generalized displacements of the nodes respectively n_1, n_2, n_3 and n_4 et N_1, N_2, N_3 and N_4 are the interpolation functions.

Such that:

$$[N] = [G][C]^{-1} \tag{4}$$

Where:

$$[G] = [1 \quad \xi \quad \eta \quad \xi\eta] \tag{5}$$

$$[C] = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \tag{6}$$

Where the one-point shape functions is expressed as:

$$\begin{cases} N_1 = \frac{1}{4}(1-\xi)(1-\eta) \\ N_2 = \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 = \frac{1}{4}(1+\xi)(1+\eta) \\ N_4 = \frac{1}{4}(1-\xi)(1+\eta) \end{cases} \quad (7)$$

2.2. Elementary stiffness matrix

The elementary stiffness matrix is calculated as:

$$\begin{aligned} [k]^e &= \int_V [B(x,y)]^T [D][B(x,y)] dV = \frac{4h}{L^2} \int_{A_{réf\acute{e}rence}} [B(\xi,\eta)]^T [D][B(\xi,\eta)] \det|J(\xi,\eta)| dA_{réf\acute{e}rence} \\ &= \frac{4h}{L^2} \int_{\xi} \int_{\eta} [B(\xi,\eta)]^T [D][B(\xi,\eta)] \det|J(\xi,\eta)| d\xi d\eta \\ &= h \int_{-1}^1 \int_{-1}^1 [B(\xi,\eta)]^T [D][B(\xi,\eta)] d\xi d\eta \end{aligned} \quad (8)$$

Where:

$$[B(x,y)] = [B_1 \quad \dots B_i \quad \dots B_n] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} = \frac{2}{L} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} & 0 \\ 0 & \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \eta} & \frac{\partial N_i}{\partial \xi} \end{bmatrix} \quad (9)$$

Hence:

$$[B(x,y)] = \frac{2}{L} \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \eta} & \frac{\partial N_4}{\partial \xi} \end{bmatrix} \quad (10)$$

For:

$$\begin{cases} x(\xi, \eta) = \sum_{i=1}^4 N_i x_i = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4 \\ y(\xi, \eta) = \sum_{i=1}^4 N_i y_i = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4 \end{cases} \quad (11)$$

The stiffness matrix for layer k of the laminates in the local laminate reference frame is expressed as:

$$[D] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \quad (12)$$

The constitutive equations of each elastic layer of the laminates in the local frame ($O_k, \vec{x}_k, \vec{y}_k, \vec{z}_k$) must be transformed into the main frame of the laminates ($O, \vec{x}, \vec{y}, \vec{z}$). The relationship of the transformation of the laminates from the local frame to the main frame is given by equation:

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{12} (\cos^4 \theta + \sin^4 \theta) \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos \theta \sin^3 \theta \\ \bar{Q}_{22} &= Q_{22} \cos^4 \theta + 2(Q_{11} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{11} \sin^4 \theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos^3 \theta \sin \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{66} (\cos^4 \theta + \sin^4 \theta) \end{aligned} \quad (13)$$

Using the material properties defined in previous equation, stiffness constants can be expressed as:

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12} \nu_{21}} \\ Q_{12} &= \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}} \\ Q_{11} &= \frac{E_2}{1 - \nu_{12} \nu_{21}} \\ Q_{11} &= \frac{E_1}{1 - \nu_{12} \nu_{21}} \\ Q_{66} &= G_{12} \end{aligned} \quad (14)$$

We can write the Jacobie matrix by the following relation:

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{L}{2} & 0 \\ 0 & \frac{L}{2} \end{bmatrix} \tag{15}$$

$$\det |J(\xi, \eta)| = \frac{L^2}{4}$$

We can thus write the elementary temperature field by the iso-parametric coordinates by the following relation:

$$T^e(\xi, \eta) = \sum_{i=1}^4 N_i(\xi, \eta) T_i = N_1(\xi, \eta) T_1 + N_2(\xi, \eta) T_2 + N_3(\xi, \eta) T_3 + N_4(\xi, \eta) T_4 \tag{16}$$

2.3. Force vector due to elementary thermal gradient

The force vector due to thermal gradient is calculated by:

$$\{f^{th}\}^e = \int_V [B(x, y)]^T [D] \{\epsilon_{th}\} dV = \frac{2h}{L} \int_{A_{reference}} [B(\xi, \eta)]^T [D] \{\epsilon_{th}\} \det |J(\xi, \eta)| dA_{reference} \tag{17}$$

$$= \frac{\Delta TLh}{2} \int_{-1}^1 \int_{-1}^1 [B(\xi, \eta)]^T [D] \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ 0 \end{Bmatrix} d\xi d\eta$$

$$[f^{th}]^e = \frac{\Delta TLh}{2} \int_{-1}^1 \int_{-1}^1 \begin{bmatrix} \frac{\partial N_1}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} \\ \frac{\partial N_2}{\partial \xi} \\ \frac{\partial N_2}{\partial \eta} \\ \frac{\partial N_3}{\partial \xi} \\ \frac{\partial N_3}{\partial \eta} \\ \frac{\partial N_4}{\partial \xi} \\ \frac{\partial N_4}{\partial \eta} \end{bmatrix} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ 0 \end{Bmatrix} d\xi d\eta$$

2.4. Formulation of the thermal behavior of a finite element laminate

The finite element formulation of the thermal equations is performed as follows:

$$\begin{aligned} \{U^i(\xi, \eta)\}^T &= \{u_i(\xi, \eta) \quad v_i(\xi, \eta)\} \\ \{U(\xi, \eta)\}^{e^T} &= \{u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3 \quad u_4 \quad v_4\} \\ \{U(\xi, \eta)\} &= \sum_e \sum_e [U(\xi, \eta)]^e \end{aligned} \quad (18)$$

$$\begin{aligned} [K(\xi, \eta)] &= \sum_e \sum_e [k(\xi, \eta)]^e \\ [F^{th}(\xi, \eta)] &= \sum_e \sum_e [f^{th}(\xi, \eta)]^e \end{aligned} \quad (19)$$

The fundamental equation of applied mathematics is given as:

$$[K(\xi, \eta)]\{U(\xi, \eta)\} = [F^{th}(\xi, \eta)] \quad (20)$$

For the thermal behavior of a laminate, the displacement field is given by the following relationships:

$$\{U(\xi, \eta)\} = [K(\xi, \eta)]^{-1} [F^{th}(\xi, \eta)] \quad (21)$$

The vector of elementary thermal forces is given by the following relation:

$$\{N^{th}(x, y)\}^e = [k(x, y)]^e \{U(x, y)\}^e - \{f^{th}(x, y)\}^e \quad (22)$$

We can build the constraint function by:

$$\sigma_{ij}^{th}(x, y) = \frac{N_{ij}^{th}(x, y)}{A_{ij}(x, y)} \quad (23)$$

The stress equations of a laminate in the local frame $(O_k, \vec{x}_k, \vec{y}_k, \vec{z}_k)$ should be transformed to the global frame of a laminate $(O, \vec{x}, \vec{y}, \vec{z})$ are given by equation (II.13):

$$\begin{aligned} \sigma_{LL}^{th} &= \sigma_{xx}^{th} \cos^2 \theta + 2\sigma_{xy}^{th} \cos \theta \sin \theta + \sigma_{yy}^{th} \sin^2 \theta \\ \sigma_{TT}^{th} &= \sigma_{yy}^{th} \cos^2 \theta - 2\sigma_{xy}^{th} \cos \theta \sin \theta + \sigma_{xx}^{th} \sin^2 \theta \\ \sigma_{LT}^{th} &= (\sigma_{yy}^{th} - \sigma_{xx}^{th}) \cos \theta \sin \theta + \sigma_{xy}^{th} (\cos^2 \theta - \sin^2 \theta) \end{aligned} \quad (24)$$

3. Numerical results and discussions

Polymerization of the laminate was carried out at a temperature of 120 °C. We want to determine the residual stresses at the operating temperature of 20 °C.

The response of thermal delation of symmetrical laminated plates. The influences of parameters such as Young's modulus ratio, fiber orientation angle and temperature variation on the thermal behavior of laminated composite material plates are examined from the several applications on laminated composite material plates.

All layers are assumed to have the same thickness, density and orthotropic material characteristics in the main axes of the material. The properties of the laminates used are:

Table 1: Mechanical and thermal characteristics of orthotropic material

$E_1 [GPa]$	$G_{12} [GPa]$	ν_{12}	$\nu_{21} = E_2 / E_1$	$\Delta T [^\circ C]$	$\alpha_{11} [1/^\circ C]$	$\alpha_{22} [1/^\circ C]$	$\alpha_{33} [1/^\circ C]$
100	$0.5E_2$	0.25	Variable	Variable	$5 \cdot 10^{-6}$	$20 \cdot 10^{-6}$	0

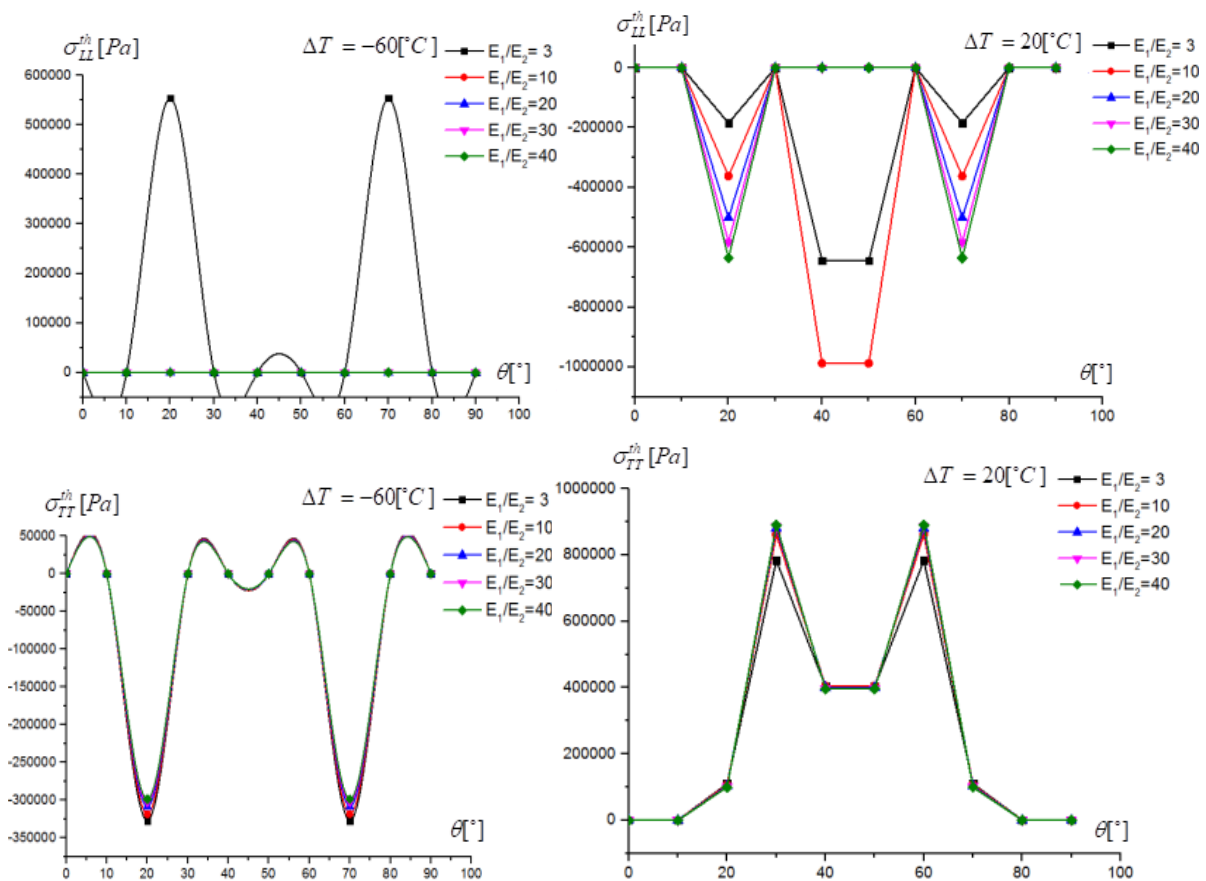
3.1. Problem of variation of the orthotropic ratio with a constant thermal gradient

Consider a three-layer symmetrical laminated composite material plate ($q \neq 0/q$), rectangular in shape subjected to a thermal gradient of constant temperature variation with a variable E_1/E_2 orthotropic ratio.

We studied the effect of the variation of the orthotropic ratio E_1/E_2 and the fiber orientation angle q on normal thermal stresses s_{LL} , s_{TT} and tangential s_{LT} with a constant thermal gradient of two cases:

1. Low temperature (cryogeny) as it finds in rooms $DT = -60^\circ C$
2. Average temperature as it finds at rooms stores medicine $DT = 20^\circ C$

Numerical results are shown in the following graphs:



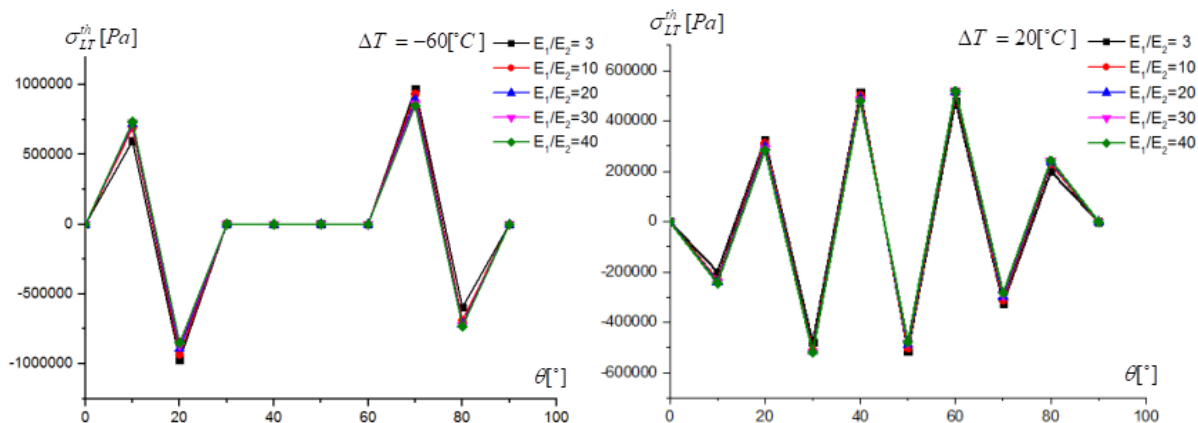


Figure 3: Variation of normal thermal stress s_{LL} , s_{TT} and tangential s_{LT} as a function of fiber orientation angle q for different orthotropic ratio value E_1/E_2 with two constant temperature variations $DT= -60^\circ\text{C}$ and $DT= 20^\circ\text{C}$.

From harsh graphical results of residual thermal stresses calculated by the finite element method,

- ❖ Variations in normal thermal stress s_{LL} as a function of fiber orientation angle q for different orthotropic ratio values E_1/E_2 .

With a temperature gradient $DT= -60^\circ\text{C}$ is lower and does not exceed 10 Pa, except for the two values of angle $q = 20^\circ$ and $q = 70^\circ$ with the orthotropic ratio $E_1/E_2=3$.

With a temperature gradient $DT= .20^\circ\text{C}$ is more symmetrically significant and shapeless compared to angle $q = 45^\circ$ and the influence of the E_1/E_2 orthotropic ratio on this constraint is very considerable. The most important constraints are found at $q = 40^\circ$ and $q = 50^\circ$ with $E_1/E_2=10$.

- ❖ The observation taken from reading these figures shows that the influence of the orientation angle of the fibers q on the normal thermal stress s_{TT} :

For $DT= -60^\circ\text{C}$ is symmetrically significant and formless of original pair function $q = 45^\circ$ with minimum thermal stresses close to 3.25 MPa at the angle $q = 20^\circ$ and the influence of the orthotropic ratio E_1/E_2 is weaker because the curves are almost identical.

For $DT= 20^\circ\text{C}$ shows that the variation with respect to the orientation angle of the fibers is an original even function $q = 45^\circ$ and the variation with respect to the orthotropic ratio E_1/E_2 is a lower one. We note that the normal s_{TT} thermal stresses are positive and maximum for $q = 30^\circ$ with the orthotropic ratio $E_1/E_2=40$.

- ❖ Analysis of the variation of the tangential thermal stress s_{LT} as a function of the orientation angle of the fibers q for different values of the orthotropic ratio E_1/E_2 with:

A temperature $DT=-60^\circ\text{C}$ shows that the variation with respect to the orientation angle of the fibers is an odd function of origin $q = 45^\circ$ and the variation with respect to the orthotropic ratio E_1/E_2 almost small and the curves are identical. We notice that the tangential thermal stress s_{LT} is much greater for $q = 20^\circ$ with $E_1/E_2= 40$.

A temperature gradient $DT= 20^\circ\text{C}$ are original odd functions $q = 45^\circ$ with this function almost identical. We notice that the tangential thermal stress s_{LT} is much greater for $q = 40^\circ$ with $E_1/E_2= 3$.

3.2. Problem of variation of the thermal gradient with a constant orthotropic ratio

Consider a plate of symmetrical laminated composite material of three layers ($q / 0/q$), rectangular in shape subjected to a thermal gradient of variable temperature variation from -60 °C up to 20 °C with a constant orthotropic ratio $E_1/E_2=25$.

We studied the effect of temperature variation DT and fiber orientation angle q on normal thermal stresses s_{LL} et s_{TT} with the orthotropic ratio $E_1/E_2=25$. The numerical results are presented in the following graphs:

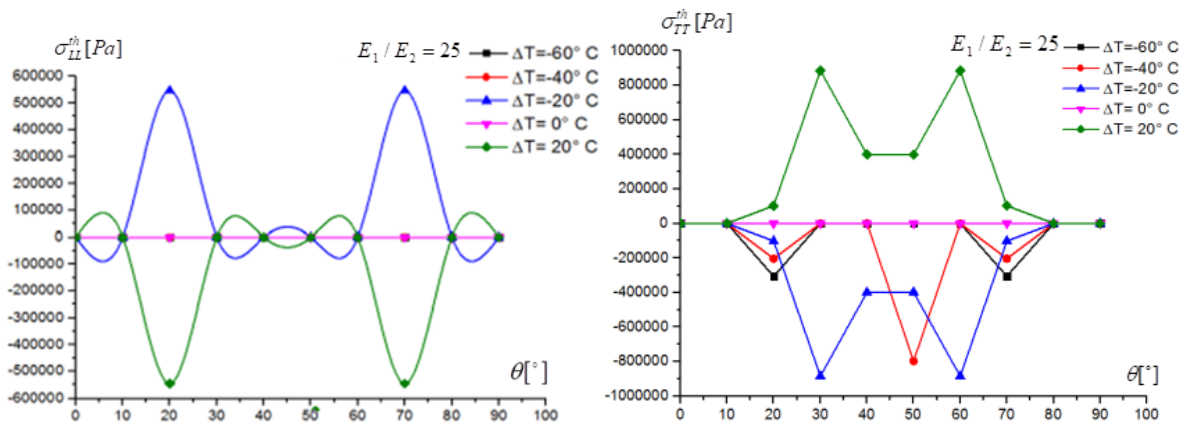


Figure 04: Variation of the normal thermal stress s_{LL} and s_{TT} as a function of the orientation angle of the fibers q for different temperature variations DT with an orthotropic ratio $E_1/E_2=25$.

According to the results obtained,

- ❖ The maximum values of the normal thermal stress s_{LL} are found at the level of $q = 20^\circ$ and $q = 70^\circ$ with $DT = -20^\circ C$ ($s_{LL} = 0.5467$ MPa).

The minimum values of normal thermal stress s_{LL} are found at the level of $q = 20^\circ$ and $q = 70^\circ$ with $DT = 20^\circ C$ ($s_{LL} = -0.5467$ MPa). Such that the maximum and minimum values of the normal thermal stress s_{LL} are varied symmetrically.

The zero values of the normal thermal stress s_{LL} are found at the level of $DT = 0$ regardless of the orientation of the fibers.

- ❖ A first observation drawn from reading these figures is that the influence of the orientation angle of the fibers q on the normal thermal stress s_{TT} is considerable in a symmetrical and formless manner with respect to the angle $q = 45^\circ$.

A second observation is that the influence of temperature variation DT on the normal thermal stress s_{TT} is considerable symmetrically and informs with respect to the temperature $DT = 0^\circ C$.

4. CONCLUSION

We studied the thermal behavior of plates made of symmetrical laminated composite materials by a numerical model of the finite elements was developed, based on the method of fine elements with rectangular elements at four nodes with two degrees of freedom per node by varying the ratio orthotropic, thermal gradient and fiber orientation angle;

These plates made of symmetrical laminated composite materials contain three layers symmetrical with respect to the layer which contains a zero-angle orientation of the fibers;

The effects of variation of various parameters such as (fiber orientation angle, orthotropic ratio, temperature variation) on the thermal behavior of plates made of symmetrical laminated composite materials are taken into account;

Normal and tangential thermal stresses are strongly influenced by variation in fiber orientation angle, orthotropic ratio and temperature variation;

The fiber orientation effect strongly influences the thermal rigidity of the plates;

Increasing the thermal gradient produces thermal stress. We can say that the thermal gradient is positive or negative due to the production of thermal stresses;

The variation in the orthotropic ratio randomly influences the thermal rigidity of plates made of symmetrical laminated composite materials;

The equality between temperatures between the two plate faces does not generate any thermal stress regardless of the temperature value.

References

- 1) Y.M., G.S. Ramtekkar and A.H. Shah, Dynamic analysis of laminated composite plates using a layerwise mixed finite element model, *Compos Struct*, 59, 237-249, 2003.
- 2) Zou, G.P., M. Naghipour and F. Taheri, A nondestructive method for evaluating natural frequency of glued-laminated beams reinforced with GRP, *Nondestructive Testing and Evaluation*, 19, 53-65, 2003.
- 3) Berthelot J-M, "Matériaux composites, comportement mécanique et analyses des structures", 2ème Edition, Masson, Paris, 1996,
- 4) J. N. Reddy, "Free vibration of antisymmetric angle-ply laminated plates including transverse shear deformation by the finite element method," *J. of Sound and Vibration*, 66 (4), 565-576 (1979).
- 5) J. N. Reddy, "A simple higher-order theory for laminated composite plates," *ASME J Appl. Mech.*, 51, 745-752 (1984).
- 6) J. N. Reddy and A. A. Khdeir, "Buckling and vibration of laminated composite plates using various plate theories," *AIAAJ*, 27(12), 1808-1817 (1989).
- 7) C. A. Shankara and N. G. Iyengar, "A C0 element for the free vibration analysis of laminated composite plates," *J. of Sound and Vibration*, 191 (5), 721-738 (1996).
- 8) M. Karama, K. S. Afaq, and S. Mistou, "Mechanical behavior of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity," *Int. J. Solids and Structures*, 40, 1525- 1546 (2003).
- 9) L. Bouyaya, "Analyse numérique du comportement dynamique de structures composites à renforcement fibreux", thèse doctorat de université de frère Mentouri Constantine, (Algérie).