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Conceptual Understanding: Students' Spatial Thinking A Cross-Sectional Study

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Abstract

The change of paradigms in the national assessment of Indonesia began in 2021 with a particular focus on teachers improving student competencies. The main issue teacher's encounter is how to improve student competencies in mathematics learning, especially in geometry. This research aims to identify spatial-thinking skills in geometry learning using Google SketchUp, based on six components of realistic geometry. This research aims to outline spatial-thinking indicators with six relevant background components of realistic geometry, and to describe students' spatial-thinking abilities in acquiring geometric comprehension by using Google SketchUp. The spatial-thinking indicators are outlined with six background components of realistic geometry by using Explanatory Factor Analysis (EFA) with JASP software version 0.15.0.0. The acquired research results show the correlations between sighting and projecting, orientating and locating, transforming, constructing and drawing, measuring and calculation, and spatial reasoning variables. Spatial-thinking indicators are outlined in six component groups of realistic geometry. There were five items success indicator in factor 1. That means that Sighting and projecting relate to success indicator. Three items success indicator in factor 2, four items success indicator in factor 3, two items success indicator in factor 4, two items success indicator in factor 5, and two items success indicator in factor 6 that met the ≥ 0.3 criterion of factor loadings were categorized as 'good' and can be used. Items with < 0.3 factor loadings were less functioning and those items were revised or eliminated. The classification naming is based on six aspects of the realistic geometry of De Moor. Students have fulfilled the six aspects of the realistic geometry of De Moor. Students have completed tasks predicting and expressing the process of drawing valid conclusions regarding a correlation of an object or a concept with another object or concept. The results can be used as the consideration for teachers in identifying students' conceptual understanding to support the improvement of student competencies.

Keywords: *Conceptual Understanding, Spatial Thinking, Geometry Learning, Design Thinking.*

INTRODUCTION

Improvements in the quality of Indonesian education continue to be implemented by the government to obtain human resources that are qualified and ready to face the challenge of the era. One of the government's current efforts is improving the national assessment at all levels of education, namely the minimum competency assessment (MCA), survey of characters, and learning environment survey [1]. According to [1], MCA is the fundamental competency assessment required by all students to develop their capacities and positively participate in

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society. Two basic competencies measured through MCA are reading literacy and mathematical literacy (numeracy). In reading literacy and numeracy, the assessed competencies encompass logic-systematic thinking skills, reasoning skills in using learned concepts and knowledge, and selecting and processing information. There were some issues encountered by teachers, in a preliminary study conducted on 10/10/21, one of which is that 60% of students did not have the minimum score of achievement (MCA) criteria set by teachers [2]. Until the writing of this paper, teachers are trying to implement MCA for getting a score that meets the desired competencies. Teachers have improved the quality of teaching by using problem-solving-based numeracy MCA [[1], [4], [5]]. Through problem-solving-based numeracy MCA, students investigate real issues which lead to the discovery of concepts, procedures, or natures/principles of mathematics.

Geometry is one of the branches of mathematics in which many of its applications are found in daily life [2]. However [2], from the result of previous studies in 2021 in Indonesia, it was shown that 65% of 11-12 years old students found difficulties in materials associated with geometry [3]. The results of interviews with teachers mention that teachers have changed the presentation of questions from routine problems into non-routine problems, including problems related to real-life and geometric investigation [1]. The hope is for students' creativity to grow so that mathematics can be brought closer to children's life [2]. However, the fact is that 60% of students failed the MCA in 2021 [4]. According to the interview results with those students, the failure is possibly due to the lack of understanding of certain concepts [[1], [4], [5], [7]]. This urgent matter motivated this study to discover students' spatial-thinking design in gaining a geometric understanding by using the support from Google SketchUp as the learning media.

Why conceptual understanding matters

The understanding of mathematical concepts is crucial in meaningful learning [[5]; [6]]. The information to be learned is arranged according to students' cognitive structures so they can relate new information with cognitive structures already familiar to them. Therefore, if a student understands new material (concepts), it means that the student understood the previous material (concepts) [[7]; [8]; [9]; [10]]. Michener states that to understand an object thoroughly, one must know 1) the object itself, 2) the relation with similar objects, 3) the relation with non-similar objects, 4) the dual relation with similar objects, 5) the relation with objects in other theories [[11]; [12]]. According to [13], conceptual understanding is understanding a concept, operation, and relation in mathematics. The authors of [14] categorize conceptual understanding into relational, instrumental, and intuitional conceptual understandings, which then each consist of four types, namely concept, operation, relation, and generalization. This categorization indicates that knowledge can be acquired by looking at the relationships between one concept and another, or it can be done by identifying the type of a problem and associating it with a problem-solving procedure, or by connecting a mathematical notation/symbol with mathematical ideas and combining it into a sequence of logical reasoning, or by using the previous knowledge that occurs automatically/directly through owned conceptual knowledge.

Understanding is divided into five categories, namely a) concept understanding, b) the understanding of principles, rules, and generalizations, c) the understanding of mathematical structures, d) the ability to create a transformation, e) the ability to follow a mindset, f) the ability to read and interpret a social issue or mathematical data [15]. These types of understanding need to be considered so misconceptions and students' lack of understanding of concepts can be minimized before moving on to the next level.

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In that order, meaningful learning can be embodied in various learning backgrounds, one of which is realistic geometry. According to De Moor [16], six aspects of realistic geometry are developed in Primary Schools: sighting and projecting, orientating and locating, transforming, constructing and drawing, measuring and calculating, and spatial reasoning. While according to [17], there are three themes: orientation and navigation, figure and construction, visualization and representation.

Thus, these types of understanding mentioned above can be made as the supporting component in meaningful learning, and are expected to improve student's understanding of one concept about another concept, so students can create an understanding scheme that is complex and long-lasting, which will be ready to be used in problem solving. Therefore, conceptual understanding must be studied as part of forming one's thinking scheme.

Spatial thinking

Spatial thinking is one of the crucial elements in geometric thinking, the component of spatial thinking are knowledge, skill, and the habit of thinking in the concept of space. By understanding the meaning of space, we can use its properties (*e.g.*, dimensionality, continuity, proximity, and separation) as a vehicle for structuring problems, finding answers, and expressing solutions. By expressing relationships within spatial structures (*e.g.*, maps, multidimensional scaling models, computer-assisted design [CAD] renderings), we can perceive, remember, and analyze the static and, via transformations, the dynamic properties of objects and the relationships between objects [18]; [19]; [20]; [4]].

Spatial thinking involves several techniques including those needed for using the concept of space, representation tools, and the reasoning process to formulate structures, for solving issues, finding the answer to a problem, and expressing it. We can use representations in a variety of modes and media (graphic [text, image, and video], tactile, auditory, kinesthetics, and olfactory) to describe, explain, and communicate the structure, operation, and function of objects and their relationships. Spatial thinking is not restricted to any domain of knowledge, although it may be more characteristic of architecture, medicine, physics, and biology, for example, than of philosophy, business administration, linguistics, and comparative literature [20].

Thinking spatially is thinking to find the meaning in shapes, sizes, locations, directions, or trajectories of objects, processes, or phenomena, or relative positions in the space of some objects, processes, or phenomena that use spatial natures as the means to structure problems (*e.g.*, maps, figures) to find answers\ and to express solutions [[21]; [22]; [23]; [24]; [25]]. The specific characteristics of spatial thinking are transformation, manipulation, and operation in representation. Operations are applied to form a concept and to arrange elements into a new concept, in which the operation varies. For example, translate or rotate objects within the space or change the spatial scale on which we operate (by zooming in or out). Or change the dimensionality of the space (collapsing from three to two dimensions). Therefore, it can be said that spatial thinking and concept understanding are two related aspects.

Transformation, manipulation, and operation in representation are used to form a concept in a situation where the formation of a concept need stages (*e.g.*, understanding how to interpret spatial representations to show sequential needs two related aspects transformation concept and representation). There are various operations of spatial thinking, so it can be said that there is no single recipe to express the way to think, *e.g.* verbally, visually, or mathematically, and there is no single way to think spatially [[20]; [26]; [27]]. This contains a meaning that

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thinking spatially demands the user to think divergently, in which every person can justify the reason for the method they use. The conceptual understanding in spatial thinking mentioned in this writing is mental activities that involve knowledge, cognitive skills, and habits of mental activities owned by someone using three components, spatial representation, transformation, and reasoning process that can be concluded through behaviors by considering the types of understanding according to [15].

Six aspects of realistic geometry according to De Moor [16] were used as the background for meaningful learning as the stage of understanding geometric concepts in this study, namely sighting and projecting, orientating and locating, transforming, constructing and drawing, measuring and calculating, and spatial reasoning to describe students' spatial thinking.

Objectives

This research aims to outline spatial-thinking indicators with six relevant background components of realistic geometry and to describe students' spatial-thinking abilities in acquiring geometric comprehension by using Google SketchUp.

METHODS

This study used mixed methods; namely quantitative data analysis supported by descriptive-qualitative information. There were 30 students in Surabaya East Java, Indonesia aged 11-12 years as the subject who provided written, interview, and questionnaire data. Google SketchUp software free version 2021 was used for supporting writing test. Google SketchUp data was used to complete the written test data. Google SketchUp data was used to clarify images that may not be clearly depicted through manual sketch drawings from written test results.

Likert scale questionnaires (see Table 1) were distributed to 30 subjects containing indicators of spatial thinking mapped with 6 components of a realistic geometric background. The questionnaire results were analyzed quantitatively using the explanatory factor analysis (EFA) test with JASP software version 0.15.0.0 (RRID: SCR_015823).

Triangulation of data through different sources is used in this research. Qualitative data were acquired from the description of the written test results of three student groups with similar answer characteristics and the interview results of three equal student groups that had met the criteria of credible data referring to similar answers from written test and interview results. Written test answers have the same meaning as interview results, students are consistent in mentioning the problem-solving process both by written test and interviews *i.e.* the performance on tasks that require mental rotation of objects, the ability to understand how objects appear at different angles, and the ability to understand how object relate to each other space, where the meaning of both data gained similarities and had been tested as valid. The researcher is a reviewer who is instrumental in translating and interpreting data generated from the respondents into meaningful information. The quantitative data as the result of (explanatory factor analysis) EFA accompanied by the data from the descriptive-qualitative analysis were used in the study in such a way that valid and reliable data were obtained. The questionnaire, written test, and interview guidelines are presented below (see Tables 1 and 2).

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Ethics statement

Ethical Approval for this study was granted by the ethics committee of Universitas PGRI Adi Buana Surabaya (approval No. 118.1/A12/LPPM/IX/2021) on 02 September 2021. Written informed consent was obtained from the parents/guardians of all students involved for the publication of de-identified data.

RESULT AND DISCUSSIONS

Quantitative data analysis

The results of the analysis of data qualitative of EFA were used to categorize items into six aspects of realistic geometry about De Moor [16]. The stages and analysis results of quantitative data are presented as follows:

Assumption test

The assumption that must be fulfilled in doing EFA is that the sample must be sufficient, which is seen from the Kaiser Meyer Olkin Test, and every variable must be correlated [28]. a. Overall MSA (measure sampling adequacy) 0.756 (≥ 0.5) Fulfilled, adequate sample (see Table 4) b. Bartlett's Test to see if variables/items are correlating, $P = 0.001$ (≤ 0.005) Fulfilled (see Table 5).

Factor loadings

Items R1 (sighting); R2 (presenting and explaining); R3 (describing how an object is positioned); R4 (determining the relative position of an object); T1 (using transformation operations, such as reflecting, rotating, or translating an object); T2 (changing perspectives (reference framework), changing orientations (mental rotation), changing shapes, changing sizes, move comprehensively, reconfiguring parts, enlarging and narrowing an object); T3 (doing transformation of concepts or terminologies from simple to complex or the transformation from one form of representation to another); P1(constructing and illustrating new ideas or concepts with old concepts known before); P2 (determining the size of an object and calculating the number of objects that will be constructed); and P3 (predicting and expressing the process of drawing valid conclusions regarding a correlation of an object or a concept with the other object or concept) were success indicators, and factor 1 (sighting and projecting); factor 2 (orienting and locating); factor 3 (transforming); factor 4 (construction and drawing); factor 5 (measuring and calculating) and factor 6 (spatial reasoning) were components of aspects realistic geometry lighting and projecting; orientating and locating; transforming; constructing and drawing; measuring and calculating; and spatial reasoning (see Table 3).

There were five items (P2, T3, T1, P1, R3) in factor 1. That means that RC1 relate to P2, T3, T1, P1 and R3 (see Table 3). Three items (R3, R4, R2) in factor 2, four items (T3, R3, R2, P3) in factor 3, two items (P1, R2) in factor 4, two items (T1, T2) in factor 5, and two items (R2, R1) in factor 6 that met the ≥ 0.3 criterion of factor loadings were categorized as 'good' and can be used. Items with < 0.3 factor loadings were less functioning and those items were revised or eliminated (see Table 6). The classification naming is based on six aspects of the realistic geometry of De Moor [16].

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Scree plot

There are six aspects of realistic geometry of De Moor [16] sighting and projecting; orientating and locating; transforming; constructing and drawing; measuring and calculating; and spatial reasoning used to categorize the items. Factor 1 sighting and projecting); factor 2 (orienting and locating); factor 3 (transforming); factor 4 (construction and drawing); factor 5 (measuring and calculating) and factor 6 (spatial reasoning).

Path diagram

The results of the path diagram show that there was a line connecting variables (dimensions) to items. The thicker the line that connects dimensions (variables) to the items the greater factor loadings, which means that items functioned optimally in explaining the dimensions paired with the items. Conversely, the thinner the line that connects dimensions (variables) to the items the less optimal the items in explaining dimensions (variables) paired with the items. On the contrary, for the line on the right side of the items, the thicker the line the greater the error (see Figure 2).

CODE NAME : AS

Circle the following statements according to your opinion!

Statement	Scale
I can present and explain objects and spatial phenomena.	1 2 3 4 5
I can describe and explain objects and spatial phenomena.	1 2 3 4 5
I can observe all the image displays.	1 2 3 4 5
I can describe (explain) the position of an object.	1 2 3 4 5
I can determine the relative position of an object.	1 2 3 4 5
I can use transformation operations such as reflecting, rotating, or translating objects.	1 2 3 4 5
I can determine the size and number of objects to be constructed.	1 2 3 4 5
I can predict and express the process of drawing valid conclusions about the correlation of another object or concept.	1 2 3 4 5

Figure 1: Questionnaire form

Table 1: Written test and interview

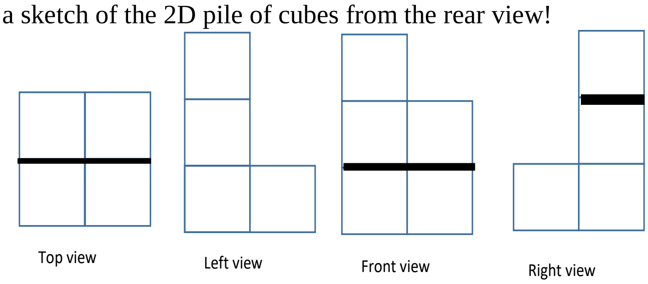
Written Test	Interview
The following shows 2D images of a pile of cubes from the top, left, front and right views. Draw a sketch of the 3D model of the pile of cubes, then draw a sketch of the 2D pile of cubes from the rear view!  Use google SketchUp to help you find the 3D model of the pile of cubes, then make the sketch in your book!	How do you get the answers? Explain every single process! (Researchers ask students, how they get the answer. It depends on the student's answers)

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Table 2: Factor loading items result

Before analysis		Factor loading result	
Component	**Success indicator**	**Component**	**Success indicator**
Sighting and projecting **(RC1)**	1. Sighting (observations) **(R1).** 2. Presenting and explaining **(R2).**	**(Factor 1)** Sighting and projecting **(RC1)**	1. Determining the size of an object and calculating the number of objects that will be constructed **(P2).** 2. Doing transformation of concepts or terminologies from simple to complex or the transformation from one form of representation to another **(T3).** 3. Using transformation operations, such as reflecting, rotating, or translating an object **(T1).** 4. Constructing and illustrating new ideas or concepts with old concepts known before **(P1).** 5. Describing how an object is positioned **(R3).**
Orienting and locating **(RC2)**	3. Describing how an object is positioned **(R3)**. 4. Determining the relative position of an object **(R4).**	**(Factor 2)** Orienting and locating **(RC2)**	1. Describing how an object is positioned **(R3)**. 2. Determining the relative position of an object **(R4).** 3. Presenting and explaining **(R2).**
Transforming **(RC3)**	1. Using transformation operations, such as reflecting, rotating, or translating an object **(T1).** 2. Changing perspectives (reference framework), changing orientations (mental rotation), changing shapes, changing sizes, move comprehensively, reconfiguring parts, enlarging and narrowing an object **(T2).** 3. Doing transformation of concepts or terminologies from simple to complex or the transformation from one form of representation to another **(T3).**	**(Factor 3)** Transforming **(RC3)**	1. Doing transformation of concepts or terminologies from simple to complex or the transformation from one form of representation to another **(T3).** 2. Describing how an object is positioned **(R3)**. 3. Presenting and explaining **(R2).** 4. Predicting and expressing the process of drawing valid conclusions regarding a correlation of an object or a concept with the other object or concept **(P3).**
Construction and drawing **(RC4)**	1. Constructing and illustrating new ideas or concepts with old concepts known before **(P1).**	**(Factor 4)** Construction and drawing **(RC4)**	1. Constructing and illustrating new ideas or concepts with old concepts known before **(P1).** 2. Presenting and explaining **(R2).**
Measuring and calculating **(RC5)**	2. Determining the size of an object and calculating the number of objects that will be constructed **(P2).**	**(Factor 5)** Measuring and calculating **(RC5)**	1. Using transformation operations, such as reflecting, rotating, or translating an object **(T1).** 2. Changing perspectives

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Before analysis		Factor loading result	
			(reference framework), changing orientations (mental rotation), changing shapes, changing sizes, move comprehensively, reconfiguring parts, enlarging and narrowing an object **(T2).**
Spatial reasoning **(RC6)**	3. Predicting and expressing the process of drawing valid conclusions regarding a correlation of an object or a concept with the other object or concept **(P3).**	**(Factor 6)** Spatial reasoning **(RC6)**	1. Sighting (observations) **(R1).** 2. Presenting and explaining **(R2).**

Table 3: The results of Kaiser-Meyer-Olkin test

	MSA (measure sampling adequacy)
Overall MSA	**0.756**
T1	**0.857**
T2	**0.884**
T3	**0.653**
R1	**0.922**
R2	**0.766**
R3	**0.727**
R4	**0.702**
P1	**0.820**
P2	**0.721**
P3	**0.609**

Table 4: The results of Bartlett's test

X^2	df	p
105.303	**45.000**	$<.001$

Table 5: Factor loadings

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Uniqueness
P1	**0.936**						**0.005**
T3	**0.755**		**0.476**				**0.087**
T1	**0.590**				**0.364**		**0.391**
P1	**0.456**			**0.798**			**0.005**
R3	**0.300**	**0.815**	**0.387**				**0.005**
R4		**0.877**					**0.106**
R2		**0.570**	**0.382**	**0.323**		**0.407**	**0.230**
P3			**0.735**				**0.403**
T2					**0.613**		**0.449**
R1							**0.604**

Note. The applied rotation method is varimax

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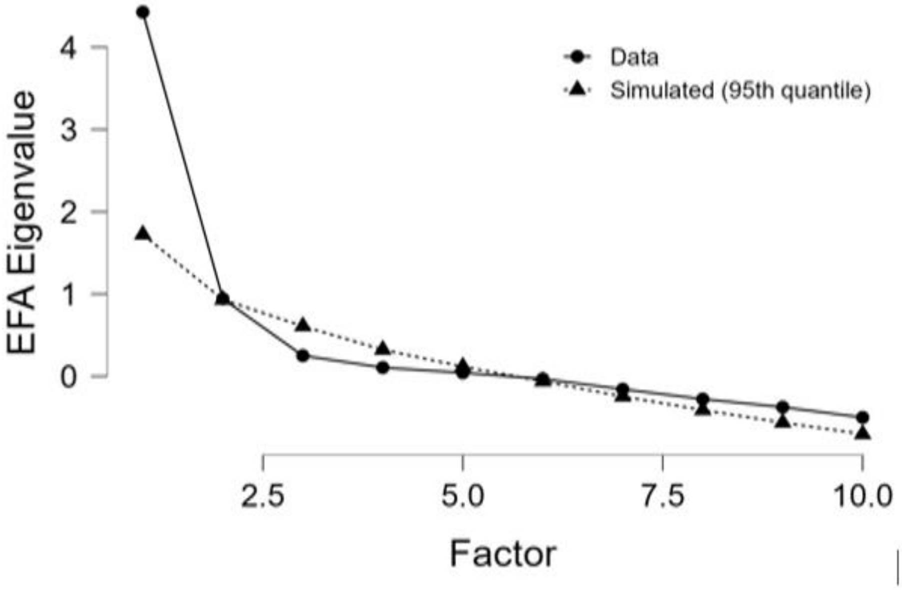

Note. EFA (Explanatory factor analysis)

Figure 2: Scree plot

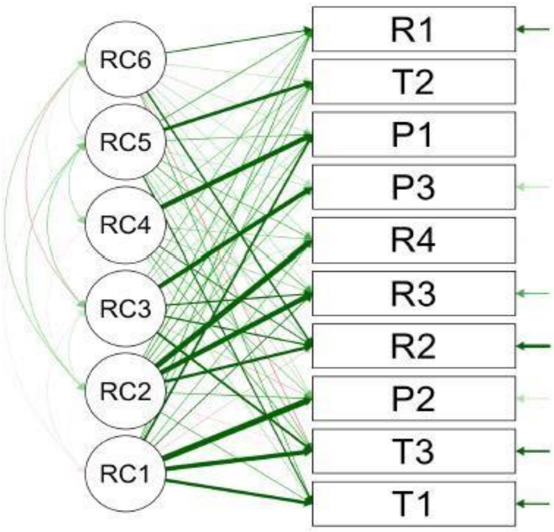

Figure 3: Path diagram

Qualitative data analysis

Subjects' interview data

The following is the characterization of the interview results with 30 subjects analyzed in a descriptive-qualitative manner. The following is one of the characterizations of the subjects' interview results.

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Table 6: The characterization of subjects' interview result

Component	Success indicator	Description of interview results
Sighting and projecting	Sighting (observations). Presenting and explaining spatial objects and phenomena.	Students are able to: The students observe all views of the picture. Subjects are oriented towards one of the "views" to find the model of the pile of cubes. Subjects are oriented towards dots, lines, and planes in determining the pile of cubes on the views (right, left, top, or front) as the embodiment of sighting on and projecting)
Orienting and locating	Describing how an object is positioned. Determining the relative position of an object.	Students are able to: Determine the position of the cube from the front to right view by orienting towards the thick line on the top view followed by sketching it. Identify the top view as the analysis material that the front-view 2D image becomes a 3D image with different heights of cube arrangement. Determine the position of the right-view cube by shifting and rotating the cube from memory in such a way until the desired sketch of 3D image is acquired.
Transforming	Using transformation operations, such as reflecting, rotating, or translating an object. Changing perspectives (reference framework), changing orientations (mental rotation), changing shapes, changing sizes, move comprehensively, reconfiguring parts, enlarging and narrowing an object. Doing transformation of concepts or terminologies from simple to complex or the transformation from one form of representation to another	Students are able to: 1. Use mental actions, such as shifting, rotating, and reflecting the cube by the front and top views by configuring its parts and rotating it to the right view. 2. Change the reference framework while determining the rear view. 3. Derive other concepts, such as directions, angles. This action can be seen when subjects said that to acquire the right view from the front view, the cube is rotated 90 degrees to the right, reflected from the previous cube, the front-view cube is parallel with the rear-view cube, and so on. Subjects acquired other concepts, such as distances, relations, alignments, intersecting lines, non-intersecting lines in space, and elementary relations in those concepts.
Construction and drawing	1. Constructing and illustrating new ideas or concepts with old concepts known before	Students are able to: 1. Construct the image with a measuring instrument without a scale.
Measuring and calculating	1. Determining the size of an object and calculating the number of objects that will be constructed.	Students are able to: 1. Determine the size of the cube to construct. 2. Calculating the number of cubes constructed on each view
Spatial reasoning	1. Predicting and expressing the process of drawing valid conclusions regarding a correlation of an object or a concept with the other object or concept	Students are able to: 1. Subjects used the "if-then" principle, namely if there is a thick line segment, then there is an arrangement of cubes with different heights. 2. Subjects concluded that there is a pile of cubes with different heights and there are five piles of rear-view cube

Result data for the written test with Google SketchUp

This test was completed using a different technique, namely a written test with Google SketchUp free version 2021. Google SketchUp data was used to complete the written test data. Google SketchUp data was used to clarify images that may not be clearly depicted through manual sketches from written test results.

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Written Test

Problems

The following shows 2D images of a pile of cubes from the top, left, front, and right views. Draw a sketch of the 3D model of the pile of cubes, then draw a sketch of the 2D pile of cubes from the rearview.

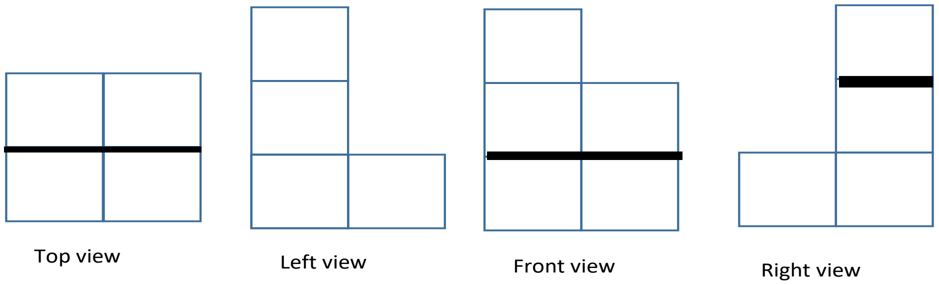

Figure 4: 2D Views

Use Google SketchUp to help you find the 3D model of the pile of cubes, then make the sketch in your book!

(1) The Characterization of the Data as the Google SketchUp Construction Result of Subject's Isometric Projection (3D)

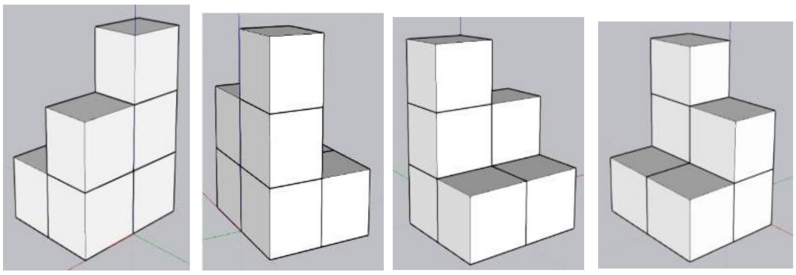

Figure 5: The Isometric Projection

(2) The Characterization of the Data as the Google SketchUp Construction Result of Subject's 2D View

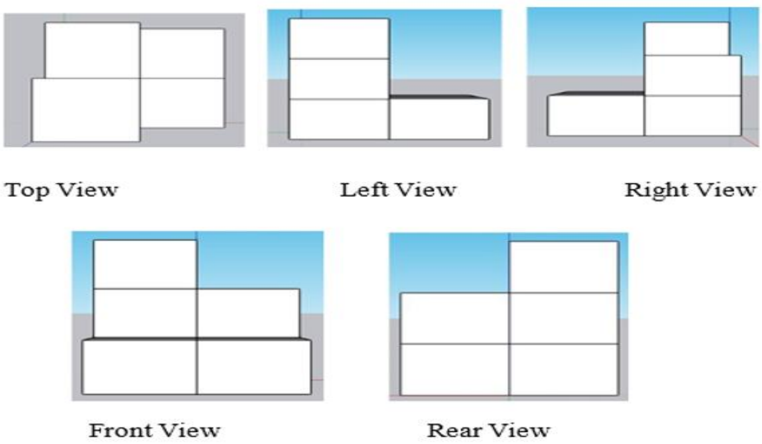

Figure 6: Google SketchUp 2D Views

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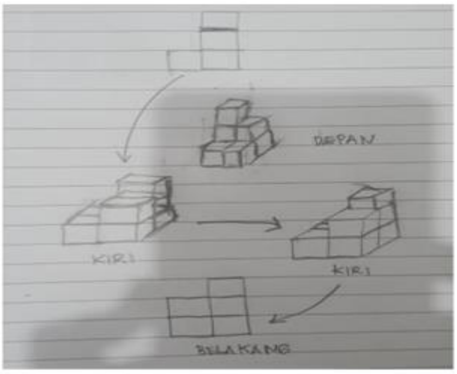
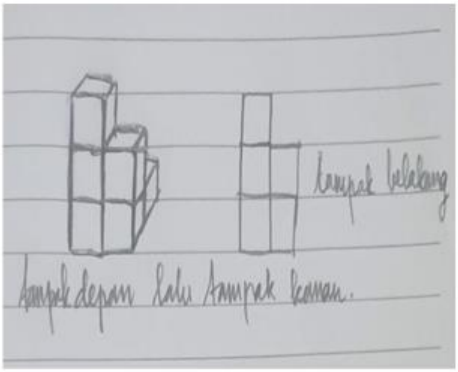
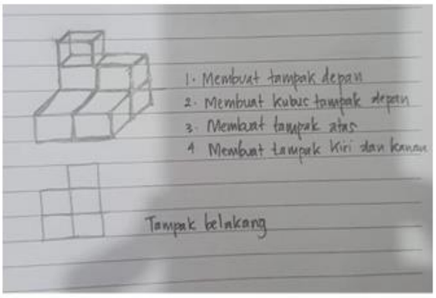
(3) The Characterization of the Data as the Sketch Construction Result of Subject's Isometric Projection (3D).



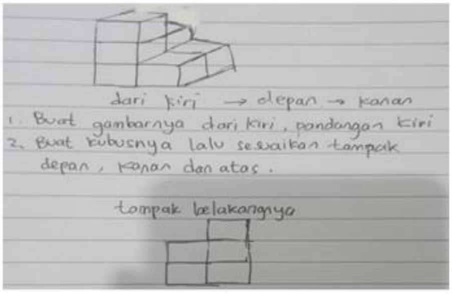
a. The sketch from the left view



b. The sketch from the right view



c. The sketch from the front view



d. The sketch from the left view

Figure 7: Drawing sketch 2D views

The results of the subjects' written test above suggest the different methods to determine the 3D model sketch of the pile of cubes, such as:

(1) The construction process of the pile of cubes is initiated by observing some given views, then one of the views is selected, namely, the front or right view left or top view [[26]; [3]; [6]; [20]].

- Depending on the left view, students sketch top-view piles of cubes. The top view shows four-unit cubes (see Figure 7a).
- From the left view, there are three piles of unit cubes on the left side (see Figure a) and one piece of cube on the right side (see Figure 7b).
- Students sketch front-view piles of cubes (see Figure 7c). From the front view, there are three piles of unit cubes on the left side (see Figure 7c) and two piles of cubes on the right side (see Figure 7d).
- From the right view (see Figure 7b), there is one pile of unit cubes on the left side (see Figure 7d) and three piles of cubes on the right side (see Figure 7b).

That means students have done sighting (observations) and then presenting and explaining spatial objects and phenomena. Sighting and projecting components were fulfilled.

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(2) Interpreting that the heights of the pile of cubes are different [[19]; [6]; [3]].

Figure 6 shows that students sketch piles of cubes, they expressed cubes without a ruler but clearly for representing piles of cubes. That means, students have determined the size of an object and calculated the number of objects that will be constructed, then predicted and expressed the process of drawing valid conclusions regarding a correlation of an object or a concept with the other object or concept. Measuring and calculating; spatial reasoning components were fulfilled.

(3) Arranging the sketch of unit cubes starting from one view, subjects took the front view [25].

Student sketch piles of cube start from left, right, and front view (see Figure 7). That means students have described how an object is positioned, then determining the relative position of an object. Orienting and locating components were fulfilled.

(4) Drawing the sketch of the unit cube model from memory by translating unit cubes according to the test instruction. Rotating the object from memory and sketching it [[20]; [3]; [19]; [6]]. Students start work from the left view, then they rotate it to another side (see Figure 6a). That means students have used transformation operations, such as reflecting, rotating, or translating an object, then changing perspectives (reference framework), changing orientations (mental rotation), changing shapes, changing sizes, moving comprehensively, reconfiguring parts, and enlarging and narrowing an object. Then Finally, they used the transformation of concepts or terminologies from simple to complex or the transformation from one form of representation to another. Transforming components were fulfilled.

(5) The results of several model sketch processes can be seen until the final results are obtained in accordance with the test instruction [26]. Students have completed the step draw sketch 2D image to 3D (see Figure 7). That means, students have done for predicting and expressing the process of drawing valid conclusions regarding a correlation of an object or a concept with the other object or concept. Spatial reasoning components were fulfilled.

(6) There are similarities in the characterization of subjects' interview results (table 7), the characterization of the Data as the Sketch Construction Result (figure 5 and Figure 6), and the written test result (figure 7).

DISCUSSION

The activities of students involved many spatial structuration and reasoning, this is consistent with [[27]; [22]; [20]. It is proven with the "if-then" implication principle, in which there will be a pile of cubes in the given model with different heights if there is a thick line segment. The measure conducted while changing the number of unit cubes on each 2D view (perspective) into many piles of unit cubes in the 3D model is the form of the implementation and development of visualization. Students should first visualize spatial transformation (starting from left to right and front view (see Figure 6)) in their heads, make predictions, and then sketch those predictions. An abstraction process is needed to realize a conceptual understanding, i.e. taking the characteristics of relevant objects and setting aside irrelevant characteristics of objects. This process requires analysis, namely sorting out "same" or "different" elements of an object and students explain their reason why an object has the same or different elements. This process has shown that there is a reasoning process to get the

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conclusion. Student activities involved transformation, they have used perspective changes (reference framework), orientations changes (mental rotation), shape changes, and size changes. Spatial thinking plays a fundamental role in how people conceive, express, and perform mathematics. The students showed they had the ability to predict and express the process of spatial reasoning i.e. the ability to imagine, visualize and differentiate objects in two or three dimensions. This shows the student's ability to understand, manipulate and modify complex data and translate concepts into concrete objects. Spatial thinking might facilitate numerical reasoning skills, including the competencies related to basic number sense and operations. It shows that spatial thinking allows students to understand the location and dimension of objects, and how different objects are related.

It also allows students to visualize and manipulate objects and shapes in their heads. Starting with sketch cubes helps students get ideas, make all the unique combinations then use them in real-world materials. The change from the application of object visualization to the development of object visualization is a form of transformation of cognitive understanding from 2D to 3D [[26]; [20]]. After examination of our results, it was found that during the completion of the model and checking if all “views” in the produced 2D model are in accordance with the 3D model [[26]; [14]; [24]; [25]]. Students have done some representation processes, i.e. they visualized 2D images as 3D images. They had to orient and locate objects, then transform from 2D to 3D. Design spatial thinking of students involved orienting and locating objects and transformation.

The subjects (students) performed a systematic sequence of work, some obtained a 3D model starting by sketching a 2D image into a 3D image from the front, right, left, and top views; or from the top, left, front, and right views; or from the right, front, left and top views with identical process characteristics. The 3D model images produced are different from each other but refer to the same 3D model, from the pre-determined criteria. This condition shows that there were various operations of spatial thinking therefore, from our results, it can be said that there is no single recipe to express the way to think verbally, visually, or mathematically, and there is no single way to think spatially [[20]; [26]; [27]; [6]]. Students had sighting (observations), presenting and explaining spatial objects and phenomena. Design spatial thinking of students involved sighting and projecting.

The Subjects (students) concluded the rear-view 2D image through systematic steps, namely observing, determining one of the views as the reference, manipulating the object by operating on it from memory (rotation, translation), [[3]; [30]]. This condition is consistent with the argument of [14] who categorizes conceptual understanding into relational, instrumental, and intuitional conceptual understandings, which then consist of four types, concept, operation, relation, and generalization.

The object operational process contains the reasoning of the “if-then” implication principle, namely if there is a thick line segment on the given model, there will be a pile of cubes with different heights, and this pile is visualized into a sketch to later conclude the rear-view image based on the acquired 3D model image. It is the “law of sufficient reason” principle of logic that something happened with a reason [29]. “If-then” is a couple of words, this language will include words related to location, distance, orientation, and direction, for example, left, right, over, under, above, below, middle, parallel, tall, and short. For older students, this language will involve the geometric vocabulary of rotations, translations, and transformations.

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This condition shows that spatial thinking is one of the crucial elements in geometric thinking made as the knowledge, skill, and thinking habit in the concept of space [[18]; [19]; [20]; [25]; [4]]. Students had sighting (observations); presenting and explaining spatial objects and phenomena; describing how an object is positioned; determining the relative position of an object; transforming objects; constructing and illustrating new ideas or concepts with old concepts known before and predicting and expressing the process of drawing. Design spatial thinking of students involved sighting and projecting; orienting and locating; transforming; construction and drawing; measuring and calculating; and spatial reasoning.

Six aspects of realistic geometry proposed by De Moor [16] can be used as the background for meaningful learning as the stage of understanding geometric concepts in this study, which encompass sighting and projecting, orientating and locating, transforming, constructing and drawing, measuring and calculation, and spatial reasoning, aimed to describe students' spatial thinking.

There is a process of understanding concepts from simple to complex, constructing knowledge by looking for the relation between one concept to another, or by identifying the type of problem then associating it with a problem-solving procedure, connecting it with mathematical ideas and combining it into a sequence of logical reasoning, or by using the previous knowledge that occurs automatically through the conceptual knowledge owned. This matter is consistent with the argument of [[14]; [25]; [4]].

Conceptual, relational, instrumental, and institutional understandings were found that were based on concept, operation, relation, and generalization [[15]; [14]] while constructing the 3D image of the pile of cubes, both in the sketch or through the Google SketchUp media.

CONCLUSIONS

The activities of students involved many spatial structuration and reasoning, which is shown in questionnaire result analysis, written result analysis, and interview result analysis. It is something about the availability of another skill. Thus, although Learner B may be less adept than Learner A with respect to rotating a visual image mentally, Learner B may be more skilled in using a different strategy that may be just as effective (*e.g.,* reasoning verbally about the relative locations of different sections of objects).

The change from the application of object visualization to the development of object visualization is a form of transformation of cognitive understanding from 2D to 3D objects. Spatial thinking can be used to externalize transformation operations (rotation, reflection, translation) by creating spatial representations in a variety of ways, forms, and sensory modalities: tactile maps or charts, auditory maps, traditional cartographic maps, two-dimensional graphics, link or flow diagrams, tree diagrams of hierarchical relationships, and three-dimensional (3-D) models.

Students rechecked their work, checking once the model was completed. Students checked all "views" to ensure their 2D model matches the 3D model. There were various operations of spatial thinking, therefore, it can be said that there is no single recipe to express the way to think verbally, visually, or mathematically, and there is no single way to think spatially. So, the process of spatial thinking comprises broad sets of interconnected competencies that can be taught and learned.

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This condition shows that spatial thinking is one of the crucial elements in geometric thinking made as the knowledge, skill, and thinking habit in the concept of space. The ultimate goal should be to foster a new generation of spatially literate students who have the habit of mind of thinking spatially, can practice spatial thinking in an informed way, and can adopt a critical stance to spatial thinking.

There is a process of understanding concepts from simple to complex, constructing knowledge by looking for the relation between one concept to another, or by identifying the type of problem then associating it with a problem-solving procedure, connecting it with mathematical ideas and combining it into a sequence of logical reasoning, or by using the previous knowledge that occurs automatically through the conceptual knowledge owned. Conceptual, relational, instrumental, and institutional understandings were found that were based on concept, operation, relation, and generalization.

Data availability

Underlying data

Zenodo: Conceptual Understanding: "What is Student Spatial Thinking Design in Geometry Learning?" https://doi.org/10.5281/zenodo.7275027 [31] This project contains the following underlying data:

- 3A. BASIC DATA (1).pdf
- 3B BASIC DATA.xlsx
- EFA RESULT NEW.pdf
- 8. QUESTIONNAIRE AND TEST RESULT.pdf

Extended data

Zenodo: Conceptual Understanding: "What is Student Spatial Thinking Design in Geometry Learning?" https://doi.org/10.5281/zenodo.7275027 [31] This project contains the following extended data:

1) PUBLISH FEE LETTER NEW.pdf
2) STATEMENT OF EVIDENCE NEW.pdf
3) 3A. BASIC DATA (1).pdf
4) 3B BASIC DATA.xlsx
5) QUESTIONNAIRE TRANSLATE INDONESIA TO ENGLISH (FREE FORM).pdf
6) ETIC DOCUMENT NEW.pdf
7) EFA RESULT NEW.pdf
8) QUESTIONNAIRE AND TEST RESULT.pdf
9) Table 2 Written Test and Interview Form.pdf

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No competing interests were disclosed.

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